Assisted History Matching and Optimization

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Some historical data

- Ph.D. at TU Delft in Applied Mathematics - Inverse modeling for air quality (2006)

- TNO/TU Delft joint position (2006 - 2013) (researcher and Asssitent Professor) – Assisted History Matching and Optimization

- Statoil, Bergen, Norway – RDI (Research, Development and Innovation)

Reservoir Engineering and Simulations

- «**Reservoir engineering** is a branch of petroleum engineering that applies scientific principles to the drainage problems arising during the development and production of oil and gas reservoirs so as to obtain a high economic recovery»

- «**Reservoir simulation** is an area of Reservoir engineering in which *computer models* are used to predict the *flow of fluids* (typically, oil, water, and gas) through *porous media*»

- Dynamics are described by the flow simulations

Petroleum vs. Atmospheric/Oceanography

Differences

Atmospheric/Oceanography

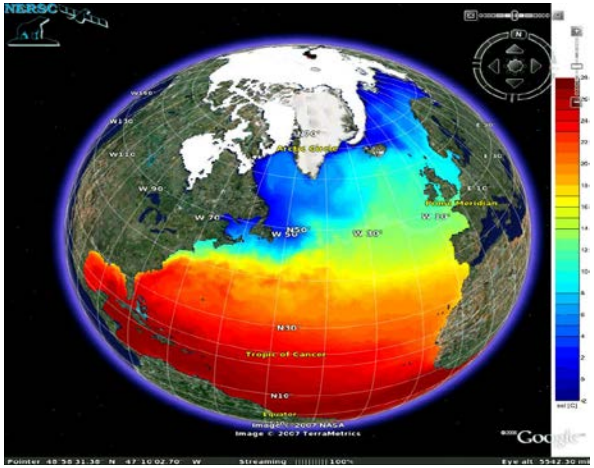
- Unstable chaotic dynamics
- State estimation problem
- Recursive problem
- 4D VAR

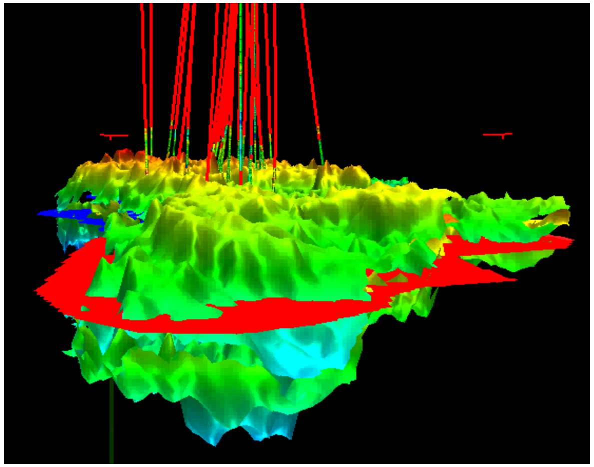
Reservoir

- Stable diffusive transport
- Parameter estimation problem
- Prediction is determined given the model
- ES

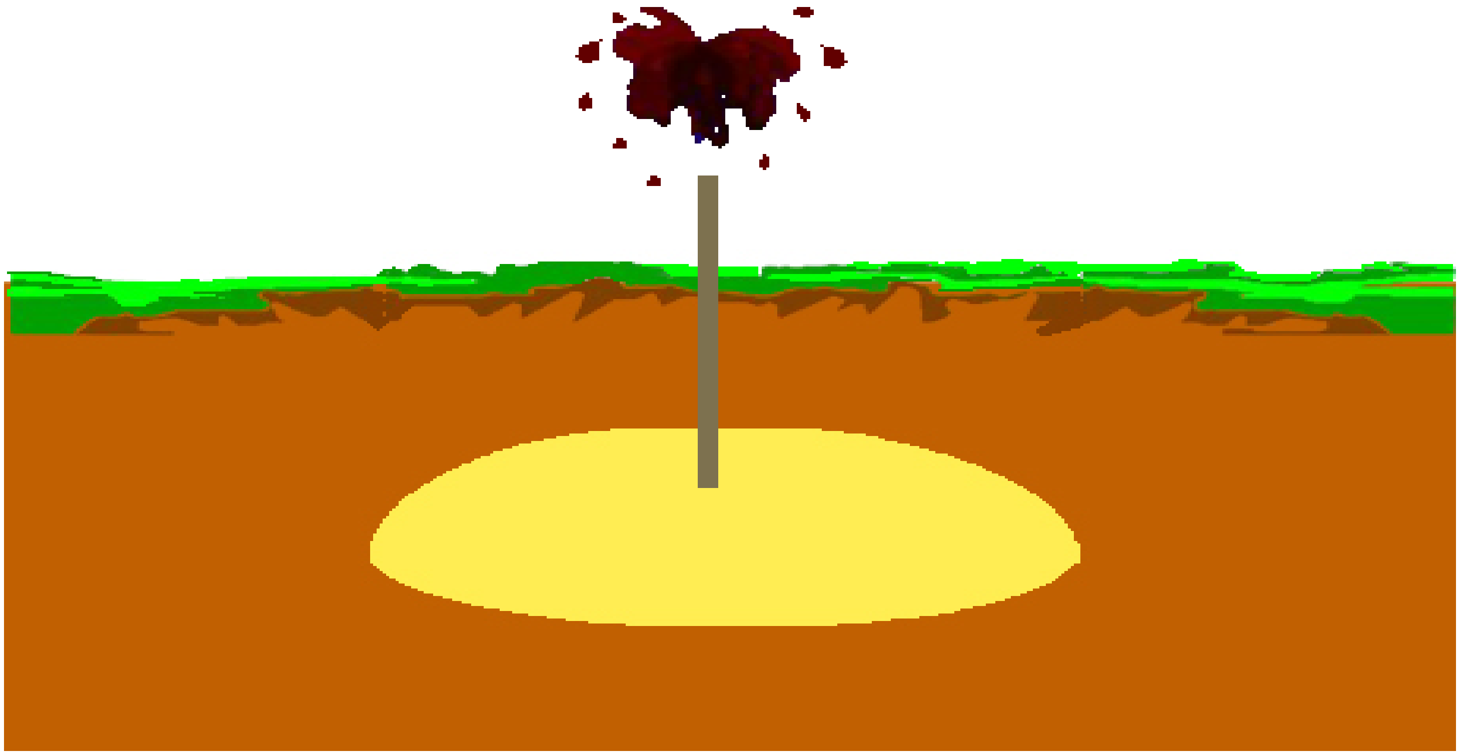
Similarities

- Dynamical prediction problem
- Uncertainty in the predictions
- Conditioning on lots of data

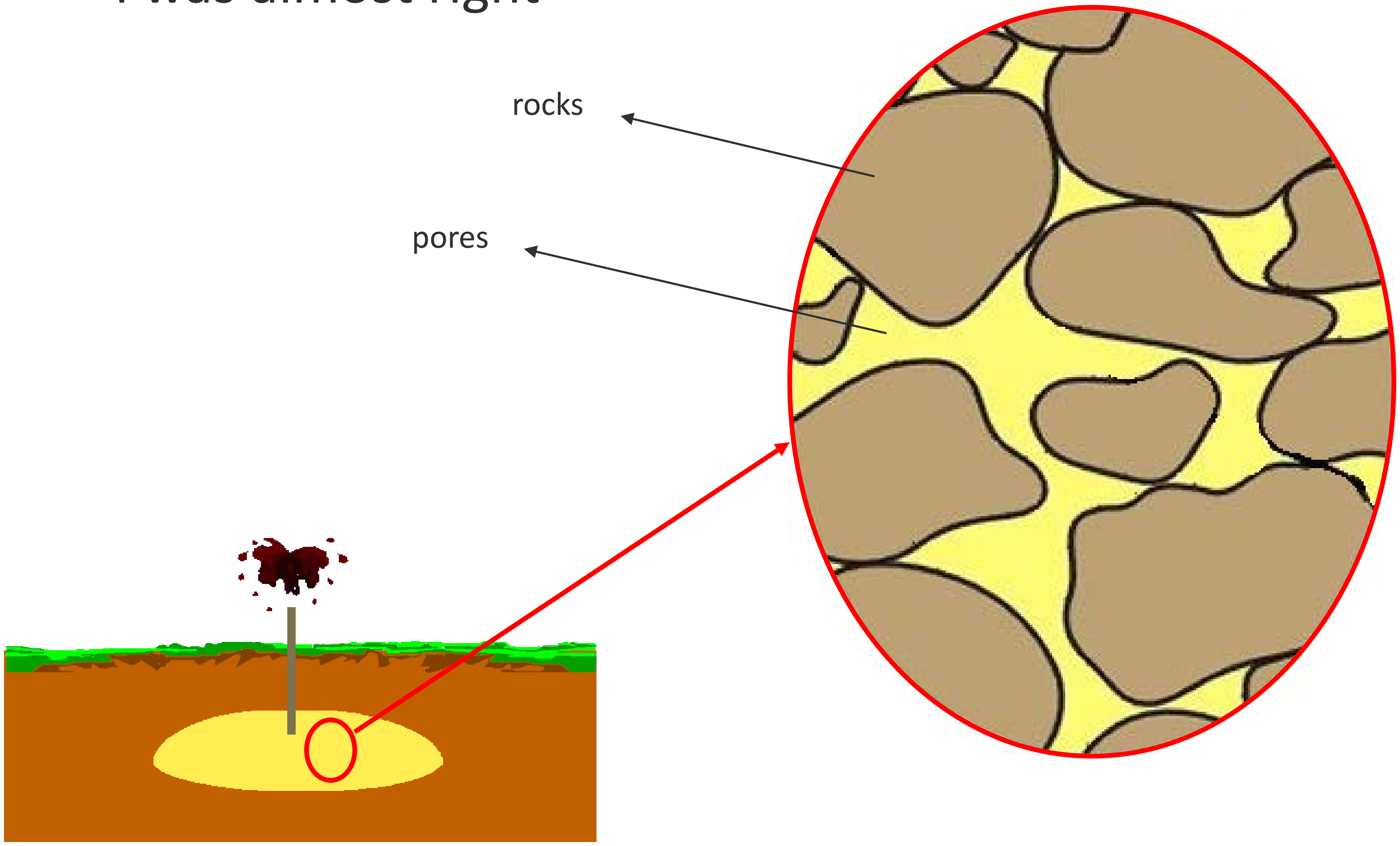




My first impression about reservoirs and trapped oil



I was almost right



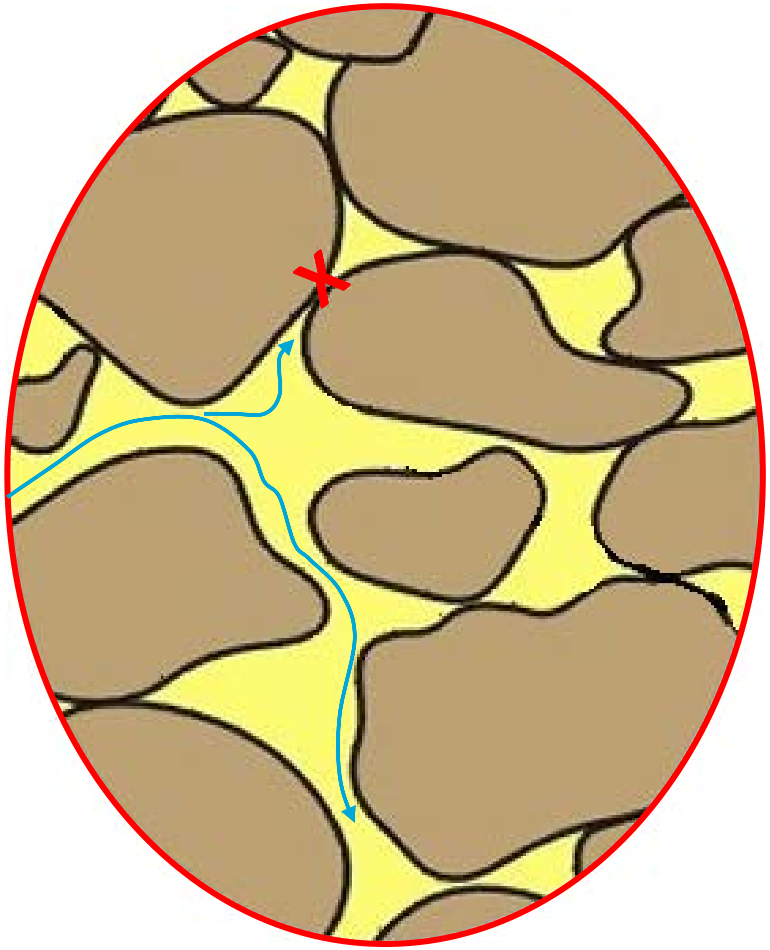
Important properties

- **porosity** – the fraction of the rock that contains fluids

- **permeability** - the ability of fluids to flow from one pore to another

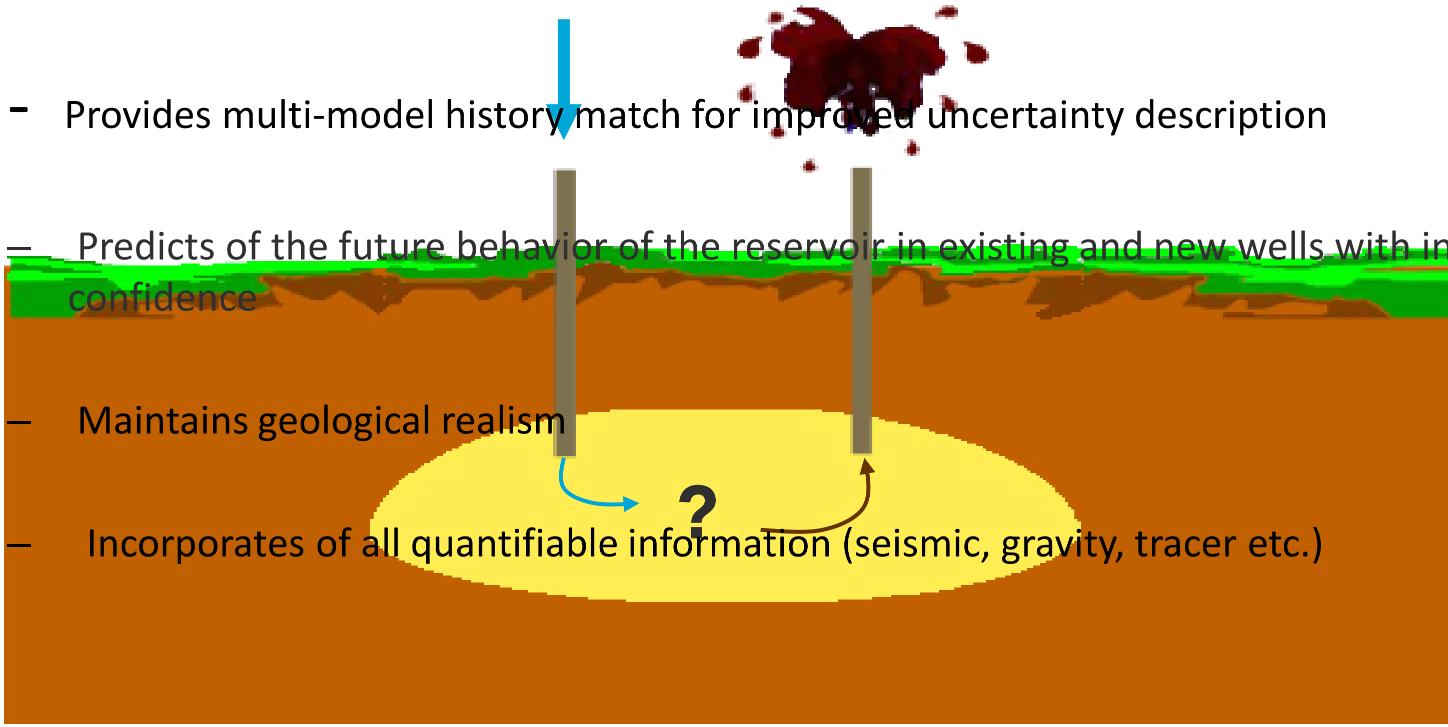
high permeable

low permeable



«Holy grail»

Probabilistic History Matching and uncertainty reduction:



- Provides multi-model history match for improved uncertainty description
- Predicts of the future behavior of the reservoir in existing and new wells with increased confidence
- Maintains geological realism
- Incorporates of all quantifiable information (seismic, gravity, tracer etc.)

Model

- The discrete model

$$\mathbf{x}_k = \mathbf{f}\left(\mathbf{x}_{k-1}, \boldsymbol{\theta}\right)$$

where

$$\mathbf{x} = [\mathbf{p}^T \ \mathbf{s}^T]^T$$

- There is no model error

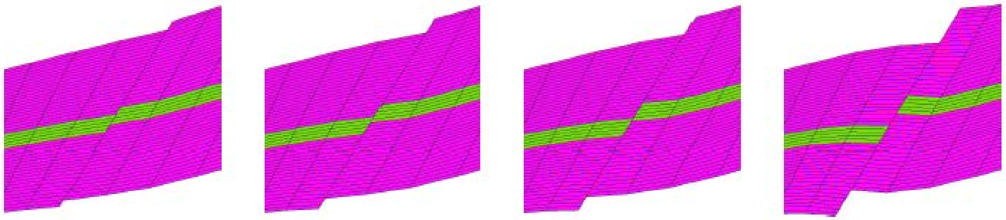
- Uncertainty in model parameters (constant) – Model calibration

- Combine the state variable and the uncertain parameters to obtain an augmented state vector

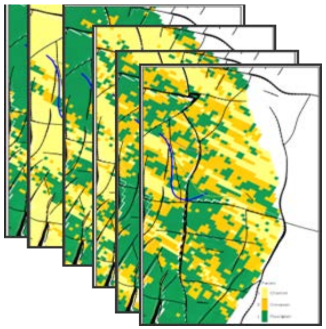
$$\hat{\mathbf{x}} = [\mathbf{x}^T \ \boldsymbol{\theta}^T]^T$$

Geological uncertainties

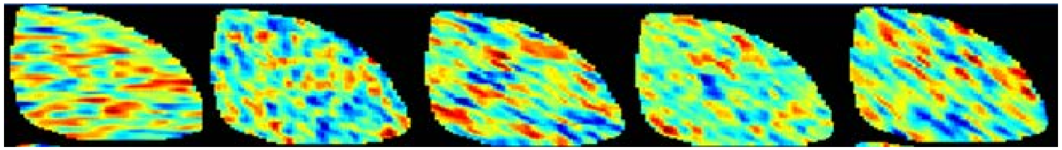
- Structural uncertainties:
    - surfaces (top/bottom)
    - faults



- Facies uncertainties



- Petro-physical uncertainties
    - Perm/poro/NTG
    - Rel.perm



Observations

- The relation between the measured data y and the state variable x is described by a nonlinear function h

$$\mathbf{y}_k = \mathbf{h}\left(\mathbf{x}_k\right) + \boldsymbol{\eta}_k$$

- Conventional observations :
    - Bottom Hole Pressure (BHP)
    - Production rates
- Unconventinal observations:
    - Seismic, tracers, gravity, satellite

Statoil

Inverse problem – Data assimilation

- Variational methods $\Leftrightarrow$ Minimization of the cost function $\Leftrightarrow$ Gradient based method $\Leftrightarrow$ Requires an adjoint

Statoil

Inverse problem – Data assimilation

The adjoint «way of life»

Inverse problem – Data assimilation

Life of the adjoint-deprived masses

Inverse problem – Data assimilation

- Variational methods ⇔ Minimization of the cost function ⇔ Gradient based method ⇔ Requires an adjoint

- Sequential methods ⇔ No adjoint required ⇔ Update every time when a measurement is available ⇔ Minimizing the variance of the estimation every analysis step.

Filter, prediction, smoother

Notations:

- $x_{0:k} = \{x_0, x_1, x_2, \dots, x_k\}$
- $y_{1:k} = \{y_1, y_2, \dots, y_k\}$
- $x^f_{k|1:k-1}$: Filter forecast at time k
- $x^a_{k|1:k}$: Filter analysis at time k
- $x^a_{k\prime|1:k}$: Smoother analysis at time k’ < k, based on observations from 1 to k.
- …

Filter, prediction, smoother

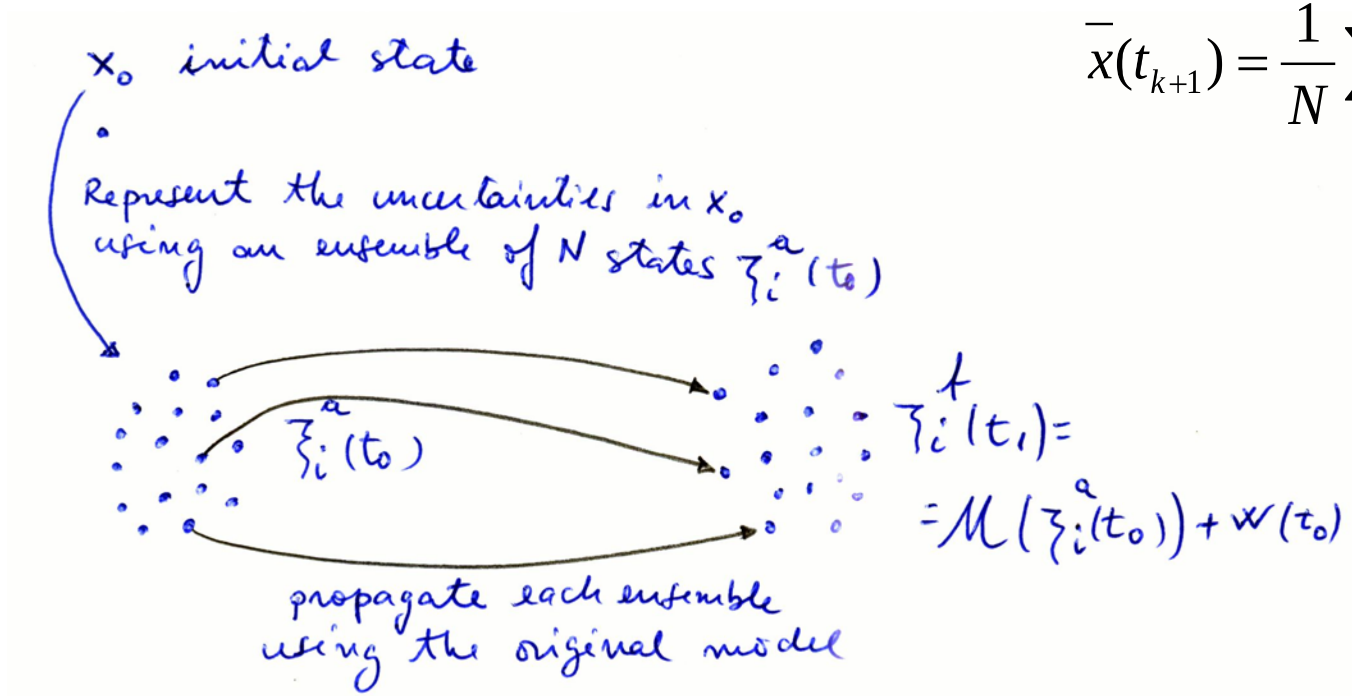
- The filtering problem:
  - consists in finding the PDF $p(x_k|y_{1:k})$
  - it is solved using a propagation/analysis generally accepted

The prediction problem

  - consists in finding the PDF $p(x_{k+l}|y_{1:k})$
  - it is solved by propagating a filter estimate by the model

Ensemble Kalman Filter

$$\bar{x}(t_{k+1}) = \frac{1}{N}\sum_{i=1}^{N} \xi_i^f(t_{k+1})$$



$$\xi_i^a(t_k) = \xi_i^f(t_{k+1}) + K(t_{k+1})[y^o(t_{k+1}) - H(t_{k+1})\xi_i^f(t_{k+1}) + v_i(t_{k+1})]$$

$$P^f(t_{k+1}) \approx P_e^f(t_{k+1}) = E[(\bar{x}(t_{k+1}) - x^f(t_{k+1}))(\bar{x}(t_{k+1}) - x^f(t_{k+1}))^T]$$

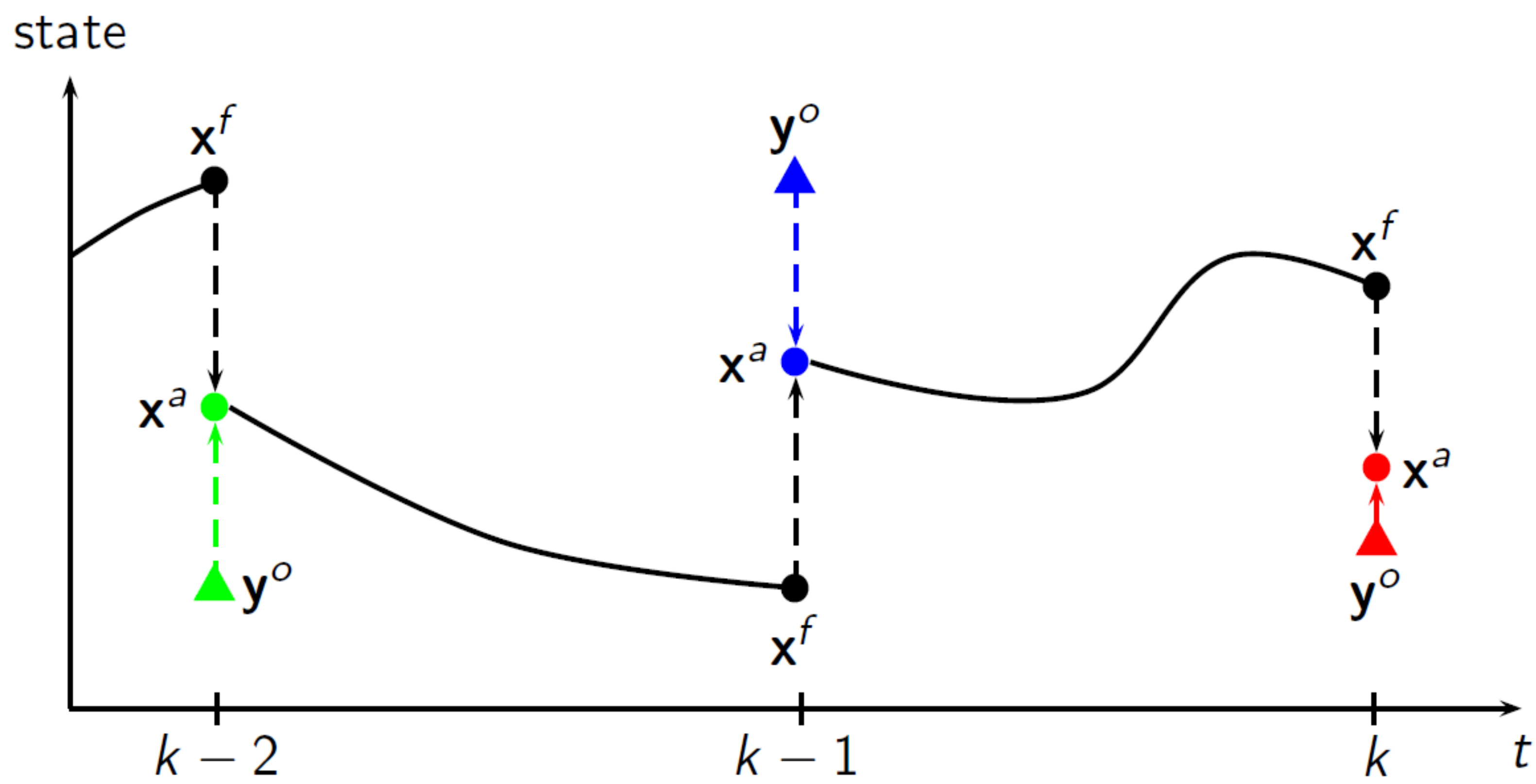
$$P^a(t_{k+1}) \approx P_e^a(t_{k+1}) = E[(\bar{x}(t_{k+1}) - x^a(t_{k+1}))(\bar{x}(t_{k+1}) - x^a(t_{k+1}))^T]$$

Ensemble smoother

- Similar to EnKF, but with a single update with all data available, i.e., no sequential data assimilation.
- Faster and easier to implement than EnKF (no restarts).
- Almost cost free (without iterations)
- Easy to implement
- Indicated for the parameter estimation problem

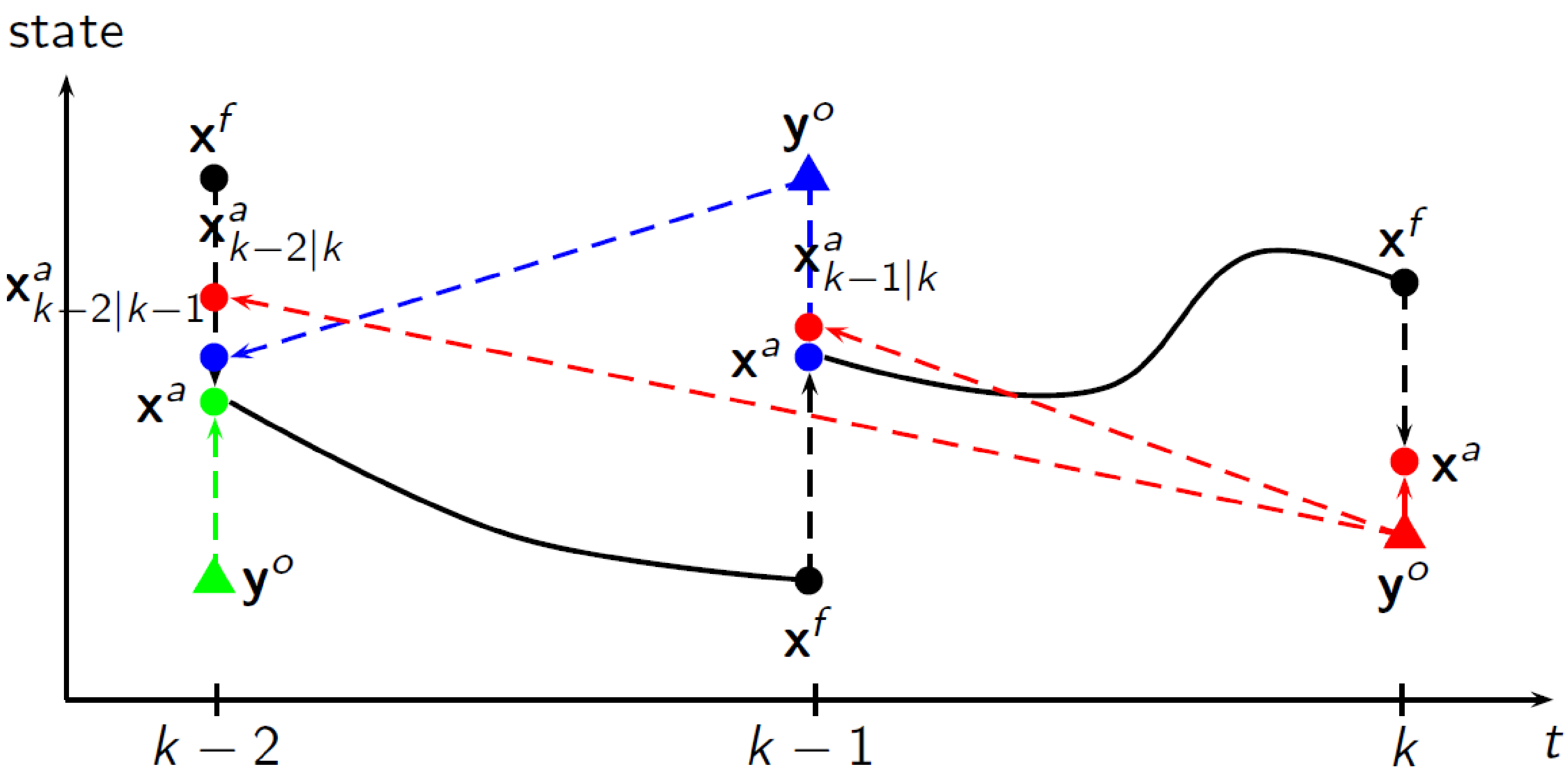
Filtering problem

- Finding the PDF $p(x_k|y_{1:k})$



Joint sequential smoothing

- Finding $p(x_{0:k}|y_{1:k})$



Multiple Data Assimilation (MDA)

Proposition

For the linear-Gaussian case, assimilating the same data multiple times with inflated covariance matrix of the measurement errors is equivalent to assimilate data only once with the original data-covariance matrix as long as the inflation factors satisfy the following condition:

$$\sum_{i=1}^{N_a} \frac{1}{\alpha_i} = 1 \qquad (\text{ex. } \alpha_i = N_a \text{ for } i = 1, \ldots, N_a).$$

Challenges

- The dimension of the state vector (dynamic and static parameters) is huge
- The forward simulation takes a lot more time that the update. (95% to 5%)
- The dimension of the ensemble goes to our infinity = 100 (magic number)
- Small number of conventional (well) measurements
- Not directly related to the parameters of interest (observability / identifiability)
- Large amount of unconventional measurements (seismic , gravity)

Challenges

1) Initial step
   - Geological uncertainties (properties, facies and structural) huge impact
   - Generation of the initial ensemble
2) Forecast step / model update
   - Need to be customized for each particular reservoir simulator
3) Update step / measurement update
   - Spurious correlations (unphysical updates or updates that are not making sense)
   - Collapse of the ensemble (divergence of the filter)

Solutions:

1) Meaningful parameterizations of the geological parameters and a tool of generating the initial ensemble
2) No solution for this (or stick with only one simulator)
3) Inflation of the variance and localization (covariance localization or local analysis)
4) Integrated workflow

Inflation trick

Covariance inflation, or ensemble inflation refers to artificial increase of uncertainty in the state estimate. It is commonly applied as follows:

$$\mathrm{P} \leftarrow \rho^2 \mathrm{P}, \;\text{ or }\; \mathrm{A} \leftarrow \rho \mathrm{A}$$

where $\rho = 1 + \delta$.

- First use in the above form - probably in Anderson (2001)
- Ott et al. (2004) use “enhanced ensemble inflation”
- Anderson (2007) introduces an adaptive algorithm
- Sacher and Bartello (2008) obtains theoretical inflation factor to compensate for finite ensemble size in the traditional EnKF
- Evensen (2009) introduces another adaptive scheme to compensate for sampling error

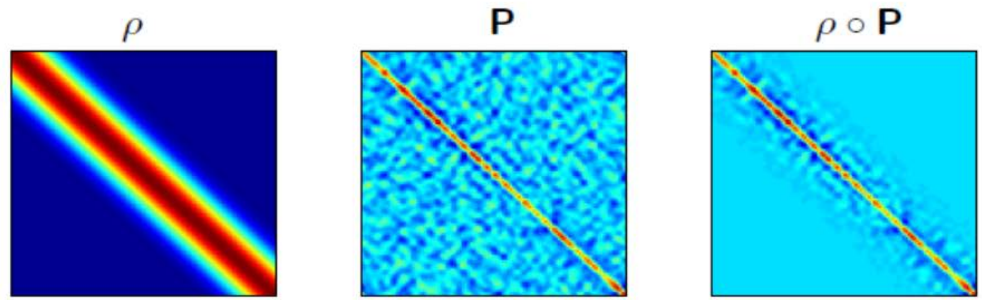
Inflation trick

- Covariance inflation is an ad-hoc modification of the method
- The inflation factor ρ is usually empirically selected
- It is often essential for preventing the (quick) filter collapse as well as for the long-term stability of the system
- In practice, a small inflation can often have a substantial positive impact on the performance of the system

Localization trick

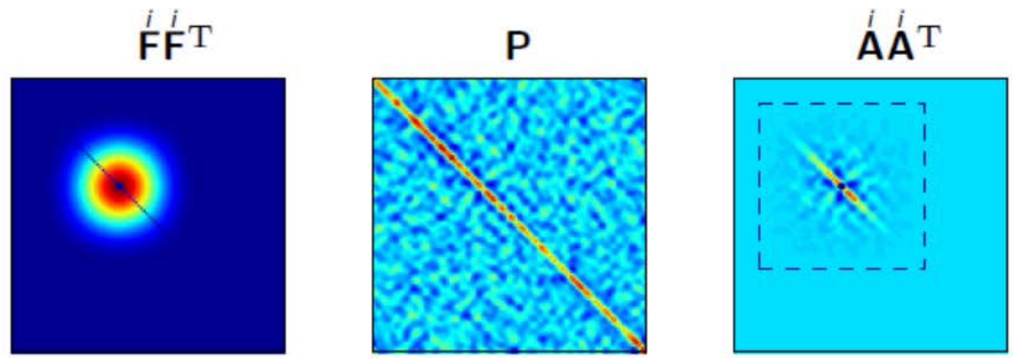
- **Covariance localization** (covariance filtering) Houtekamer and Mitchell (2001); Whitaker and Hamill (2002)

$$\mathbf{P} \to \rho \circ \mathbf{P}$$



- **Local analysis** (Evensen, 2003; Anderson, 2003; Ott et al., 2004; Hunt et al.,

$$i : \mathbf{A} \to \overset{i}{\mathbf{A}}$$



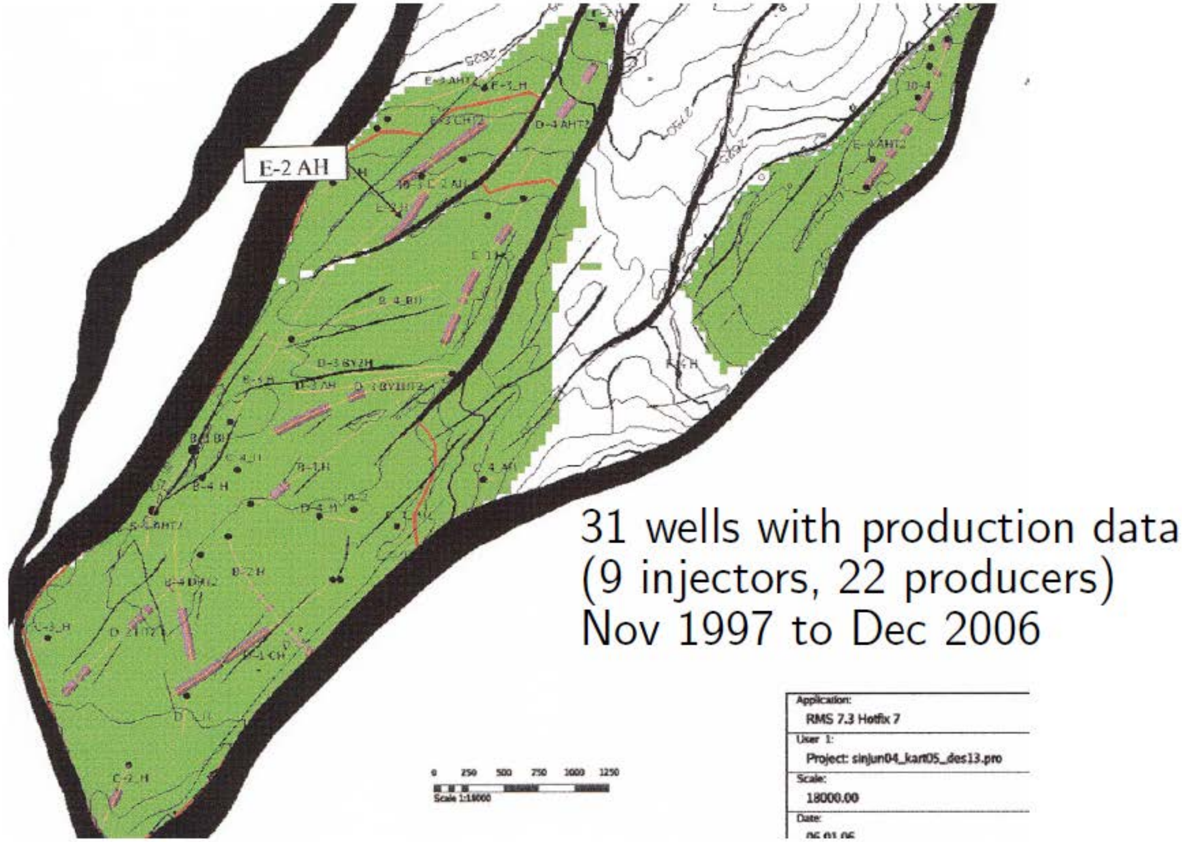
Localization trick

- Covariance localisation: Hamill and Whitaker (2001); Houtekamer and Mitchell (2001)
- Local analysis: Evensen (2003); Anderson (2003); Ott et al. (2004)
- Smooth tapering in parallel ESRFs: Hunt et al. (2007)
- Adaptive methods: Anderson (2007); Bishop and Hodyss (2007, 2009)
- Dean Oliver 2012, bootstarping of the Kalman gains
- Al Reynolds 2010 , adaptive choice of the lenght

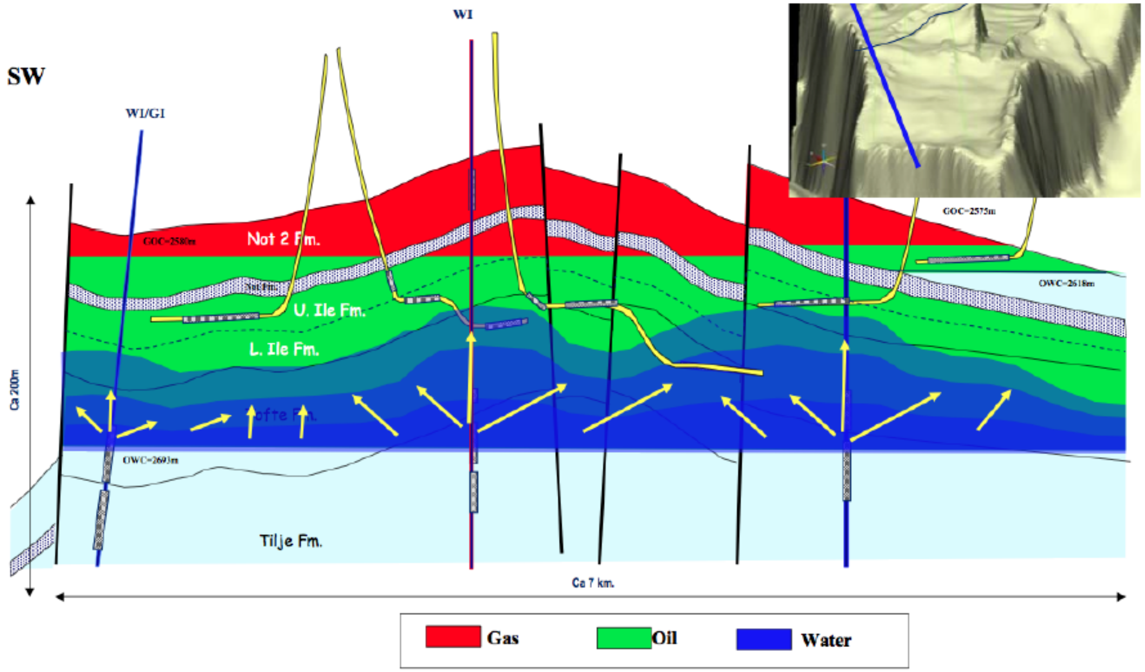
Localization trick

- Localization in the EnKF is an ad-hoc modification of the analysis scheme
- One must use localization if the ensemble size is smaller than the model subspace dimension
- Localization makes the analysis schemes suboptimal and therefore inconsistent (sometimes - strongly) in regard to estimation of posterior covariance
- Localization makes it possible to recover the modal structure from observations even with a rank-deficient ensemble

Norne field


E-2 AH
31 wells with production data
(9 injectors, 22 producers)
Nov 1997 to Dec 2006
Application:
RMS 7.3 Hotfix 7
User 1:
Project: sinjun04_kart05_des13.pro
Scale:
18000.00
Date:
Scale 1:18000

Norne field



Horizontal wells, gas cap, oil zone, water zone, faults, vertical barriers, variation in fluid contacts.

Norne field

Uncertain parameters:

- porosity, permeability, NTG, MULTZ (grid-based properties)
- MULTFLT - fault transmissibility multipliers (53 parameters)
- Rel. perm – End-point water and gas relative permeability of four zones (8 parameters)
- OWC Initial oil-water contact depth in 4 regions

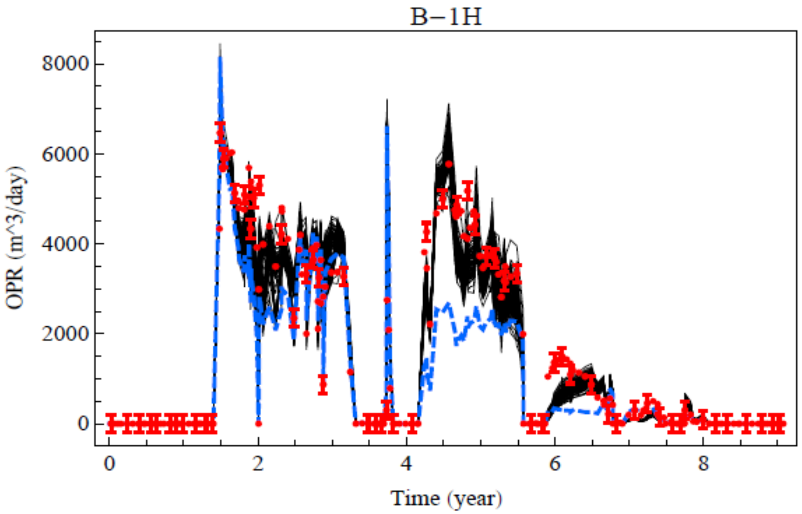
In total we have 150000 uncertain parameters

Measurements used in the AHM process:

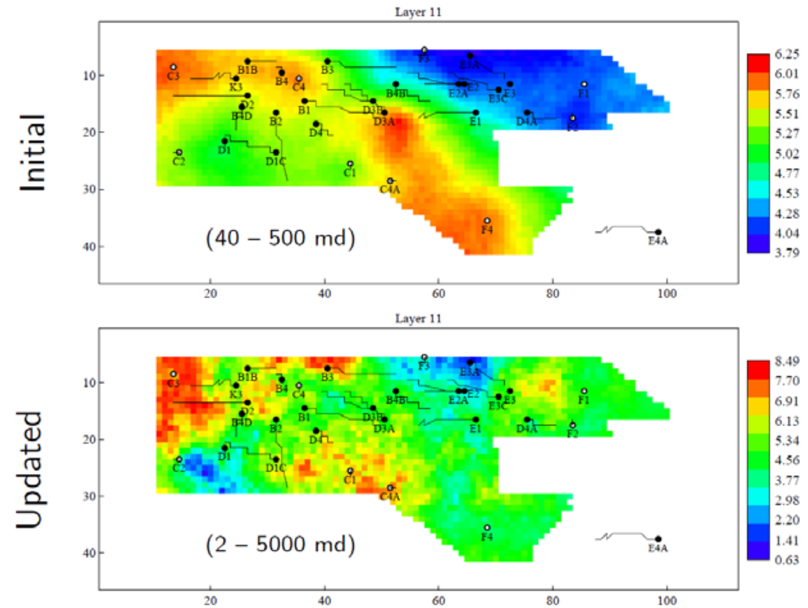
- Water injection rate
- Gas injection rate
- Oil production rate
- Water production rate
- Gas production rate
- RFT pressure

In total we have 2000 measurements

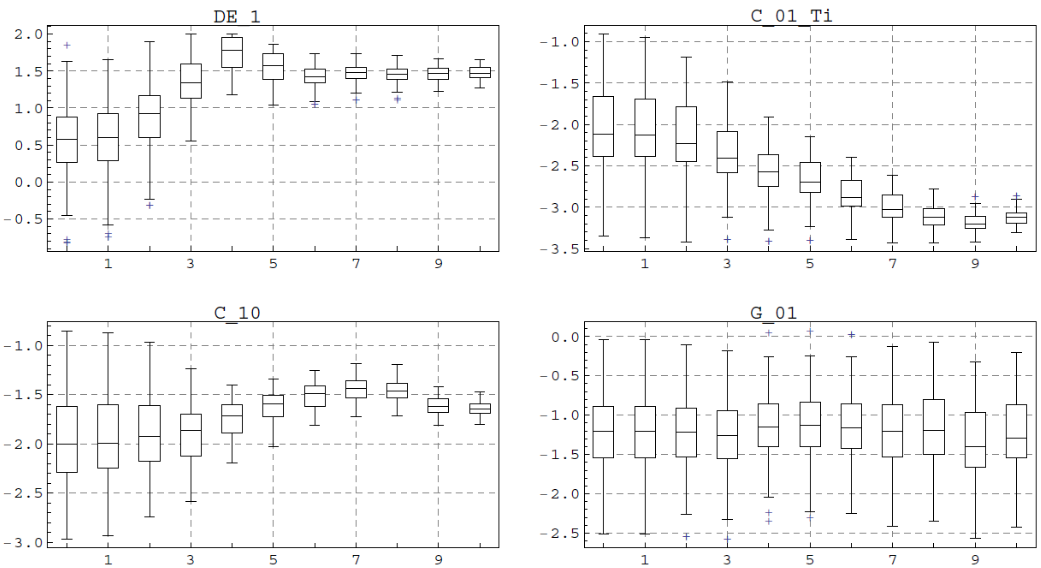
Oil production rates matching



Field permeabilty updates



Fault transmisibility multiplier



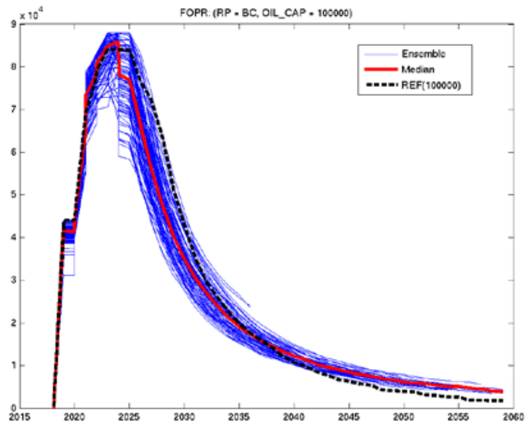
Multiple realizations / The ensemble

- How to handle multiple realisations in daily operations
  - Model QC
  - Management decisions (predictions)
  - Well planning

- Where to start and finish with parameterisation & uncertainty

Reservoir management (cont'd)

Optimize drilling priority using Fast Model Update (FMU) and Ensemble Reservoir Tool (ERT)



- Not only on the base case and also tacking the dynamics into account – link with a NPV maximization (optimization tool)

- The key functions are the ability to open a well at a varying date in the schedule file.

- Ability to model constraints between the dates (e.g. minimum of 30 days between well1 and well2 opening due to drilling time).

Reservoir management

Well placement under geological uncertainty (replicate geo-steering)

- The aim is to optimize the planned well path under this geological uncertainty
- This may have a significant impact of NPV estimations
- We will not actually do any geo-steering, but will try to adjust a planned well path in the simulation model in a way consistent with geo-steering

Wells “falling out” of the analysis (placed in non-net)

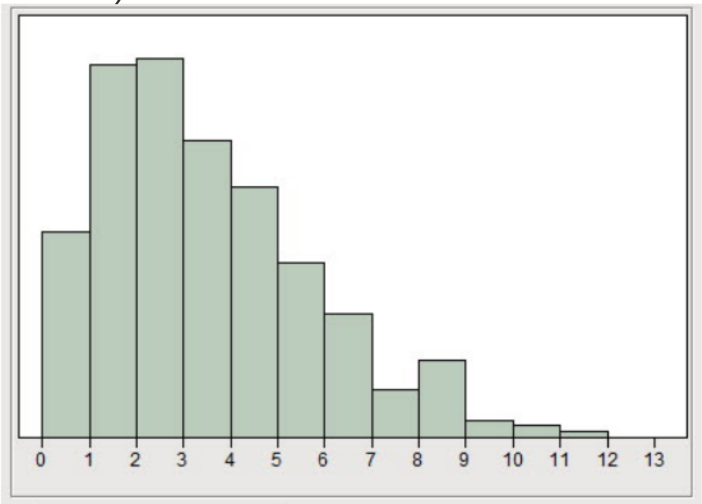
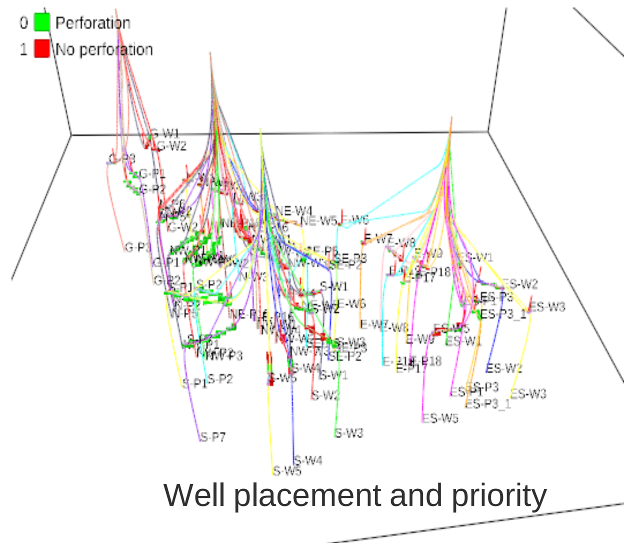


Figure 12.26 Number of excluded wells. This figure displays the relative frequency of having a certain number of wells excluded in the Eclipse runs due to structural uncertainties and facies uncertainties.

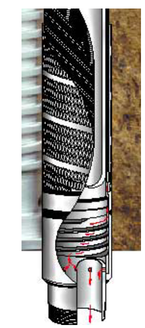
Robust optimization

- Robust optimization (multiple realizations)

- Optimization (base case)
  - ICD, ICV (choke control)
  - Well placement
  - Number of wells
  - Production strategies
  - Drilling priority

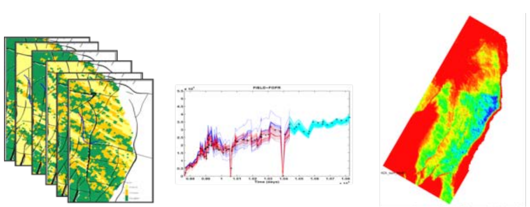


Well placement and priority



ICD

- Robust optimization (multiple realizations)
  - Optimization under geological uncertainty



Facies realizations

Production forecast

Estimated probability of finding remaining oil

- IOR and accelerated recovery due to improved and optimized production strategy

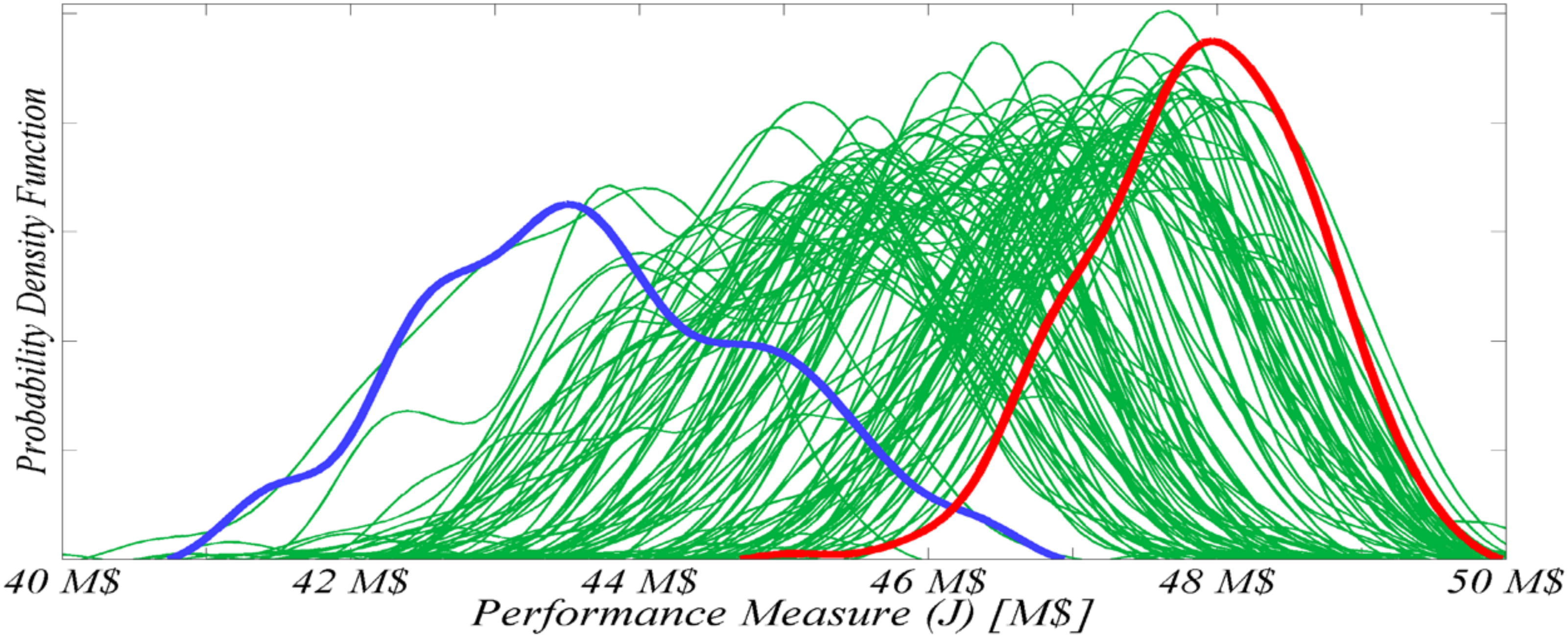
Robust optimization

- Three control strategies applied to a set of realizations.

Reactive Control:
- Based on a single realization.
- What we get from the today workflows.

Nominal optimization:
- Optimization on one realization and applying controls on all other realizations.
- Repeat optimization procedure for all realizations and reapply.

Robust Optimization:
- Maximizing expected NPV over all realizations.



Run movie

There's never been a better time for good ideas

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Filter, prediction, smoother

- Any other type of PDF defines a smoothing problem. For example:
  - $p(x_{0:k}|y_{1:k})$: joint smoothing problem of the fixed interval type
  - Can be used for estimation of chemical reaction rates based on observations from a field campaign

  - $p(x_{k-L:k}|y_{1:k})$: joint smoothing problem of the fixed-lag type
  - It is use for Reanalysis

  - $p(x_{k'}|y_{1:k})$, with $1 \leq k' < k$: Marginal or fixed-point smoothing problem
  - Can be used for the emission source of a pollutant of interest

Smoothing algorithms

- Different types of smoothing problems

- Since some information must be propagated backward in time, no obvoius algortihm prevails

- Hence, a large number of different algorithms, compared with the filtering and prediction problems

Sequential smoother

- It runs along with a filter
- The really new concept in the smoother, with respect to the Kalman filter, is the use (explicit or implicit) of the following cross-covariance matrices:

$$\mathbf{P}^{fa}_{k,i|k-1} = E[\epsilon^{f}_{k|k-1}\epsilon^{a}_{i|k-1}{}^{T}].$$

$$\mathbf{P}^{aa}_{k,i|k} = E[\epsilon^{a}_{k|k}\epsilon^{a}_{i|k}{}^{T}].$$

When the filter process the observations at time k to update the state at time k, the smoother uses the same observations to update the states at times i < k. So, the above define are the cross-covariances between the smoother forecast and the analasis at time i with the filter analysis at time k.

Sequential smoother

The sequential smoother can be derived by extending the filter state vector with past states, e.g.:

$$\mathbf{x}^a_{0:k-1} = \begin{pmatrix} \mathbf{x}^a_{k-1|k-1} \\ \mathbf{x}^a_{k-2|k-1} \\ \vdots \\ \mathbf{x}^a_{0|k-1} \end{pmatrix}$$

Sequential smoother

The extended covariance matrix has the form

$$
\mathbf{P}^{a}_{0:k-1|k-1} = \begin{pmatrix}
\mathbf{P}^{a}_{k-1|k-1} & \mathbf{P}^{aa}_{k-1,k-2|k-1} & \mathbf{P}^{aa}_{k-1,k-3|k-1} & \cdots & \cdots & \mathbf{P}^{aa}_{k-1,0|k-1} \\
\mathbf{P}^{aa}_{k-2,k-1|k-1} & \mathbf{P}^{a}_{k-2|k-1} & \mathbf{P}^{aa}_{k-2,k-3|k-1} & & & \mathbf{P}^{aa}_{k-2,0|k-1} \\
\mathbf{P}^{aa}_{k-3,k-1|k-1} & \mathbf{P}^{aa}_{k-3,k-2|k-1} & \mathbf{P}^{a}_{k-3|k-1} & & & \vdots \\
\vdots & & & \ddots & & \vdots \\
\vdots & & & & \ddots & \mathbf{P}^{aa}_{1,0|k-1} \\
\mathbf{P}^{aa}_{0,k-1|k-1} & \mathbf{P}^{aa}_{0,k-2|k-1} & \cdots & \cdots & \mathbf{P}^{aa}_{0,1|k-1} & \mathbf{P}^{a}_{0|k-1}
\end{pmatrix}
$$

Sequential smoother

The extended model is obtained:

$$\mathbf{M}_{0:k} = \begin{pmatrix} \mathbf{M}_{k-1,k} & 0 & \cdots & 0 \\ \mathbf{I} & 0 & \ddots & \vdots \\ 0 & \mathbf{I} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{I} \end{pmatrix}$$

Sequential smoother

The propagated state is:

$$\mathbf{x}^f_{0:k} = \mathbf{M}_{0:k}\mathbf{x}^a_{0:k-1} = \begin{pmatrix} \mathbf{M}_{k-1,k}\mathbf{x}^a_{k-1|k-1} \\ \mathbf{x}^a_{k-1|k-1} \\ \mathbf{x}^a_{k-2|k-1} \\ \vdots \\ \mathbf{x}^a_{0|k-1} \end{pmatrix} = \begin{pmatrix} \mathbf{x}^f_{k|k-1} \\ \mathbf{x}^a_{k-1|k-1} \\ \mathbf{x}^a_{k-2|k-1} \\ \vdots \\ \mathbf{x}^a_{0|k-1} \end{pmatrix}$$

Ensemble smoother

- The cross-coavriance matrix

$$\mathbf{P}^{fa}_{k,i|1:k-1} = E[\epsilon^{f}_{k|1:k-1} {\epsilon^{a}_{i|1:k-1}}^{T}]$$

- Ensemble-based approaximation

$$\begin{aligned} \mathbf{P}^{fa}_{k,i|1:k-1} &= \frac{1}{N_e - 1} \sum_{m=1}^{N_e} (\mathbf{x}^{f,m}_{k|1:k-1} - \hat{\mathbf{x}}^{f}_{k|1:k-1})(\mathbf{x}^{a,m}_{i|1:k-1} - \hat{\mathbf{x}}^{a}_{i|1:k-1}) \\ &= \mathbf{X}^{f}_{k|1:k-1} {\mathbf{X}^{a}_{i|1:k-1}}^{T}. \end{aligned}$$

Ensemble smoother

- The EnKF analysis

$$
\begin{aligned}
(\mathbf{X}^f)_m &= \frac{\mathbf{x}^{f,m} - \hat{\mathbf{x}}^f}{\sqrt{N_e - 1}}, \\
\gamma^m &= (\mathbf{H}\mathbf{X}^f)^T[(\mathbf{H}\mathbf{X}^f)(\mathbf{H}\mathbf{X}^f)^T + \mathbf{R}]^{-1}(\mathbf{y}^{o,m} - \mathbf{H}\mathbf{x}^{f,m}), \\
\mathbf{x}^{a,m} &= \mathbf{x}^{f,m} + \mathbf{X}^f\gamma^m.
\end{aligned}
$$

The filter corrections are linear combinations of ensemble perturbations

- The smoother analysis

$$
\begin{aligned}
\mathbf{K}_{i|1:k} &= (\mathbf{H}_k\mathbf{P}^{fa}_{k,i|1:k-1})^T\mathbf{G}_k^{-1} \\
&= \mathbf{X}^a_{i|1:k-1}(\mathbf{X}^f_{k|1:k-1})^T\mathbf{H}_k{}^T\mathbf{G}_k^{-1}.
\end{aligned}
$$

$$
\mathbf{x}^{a,m}_{i|1:k} = \mathbf{x}^{a,m}_{i|1:k-1} + \mathbf{X}^a_{i|1:k-1}\gamma^m.
$$

The smoother corrections are linear combinations of ensemble perturbations

Sequential smoother

- Almost cost free (without iterations)
- Easy to implement
- Easy to switch from fixed-interval to fixed-lag to fixed-point types
- Indicated for the parameter estimation problem
- Does not give a better estimation than the EnKF of the current time
- The choice of using the smoother is tailored to the specific field of application (based on practical considerations)

Robust reservoir management

- Three control strategies applied to a set of 100 realizations.
- All curves represent PDFs from 100 realizations

Robust Optimization:

- Maximizing over all individual realizations.