On the use of Lagrangian Coherent Structures in direct assimilation of ocean tracer images

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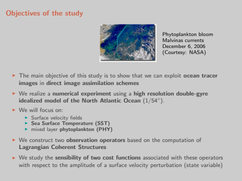
Objectives of the study

Phytoplankton bloom
Malvinas currents
December 6, 2006
(Courtesy: NASA)

- The main objective of this study is to show that we can exploit **ocean tracer images** in **direct image assimilation schemes**
- We realize a **numerical experiment** using a **high resolution double-gyre idealized model of the North Atlantic Ocean** ($1/54^\circ$).
- We will focus on:
  - Surface velocity fields
  - **Sea Surface Temperature (SST)**
  - mixed layer **phytoplankton (PHY)**
- We construct two **observation operators** based on the computation of **Lagrangian Coherent Structures**
- We study the **sensibility of two cost functions** associated with these operators with respect to the amplitude of a surface velocity perturbation (state variable)

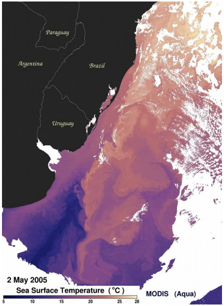


- This talk presents an impact study based on a numerical experiment that shows the potential of high resolution ocean tracer images for data assimilation in meso-scale models
- Direct assimilation of images into geophysical fluid models is a scientific challenge suggested few years ago by François-Xavier Le Dimet (INRIA MOISE/LJK, Grenoble, France). As many challenges, it opened a lot of questions but many of them are still not investigated
- The work presented here was done at INRIA and LEGI, France. It was financed by a fund of the French Research Agency (ANR).

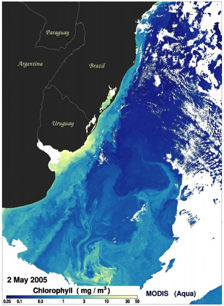
Outline

Direct Image Assimilation

Motivations



Sea Surface Temperature

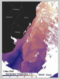


Ocean Color

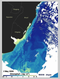
Convergence of the southward flowing Brazil and northward flowing Malvinas currents
May 2, 2005
AQUA MODIS
(Courtesy: NASA)

- **Ocean tracer images** contain structured information that should be exploited
- Ocean color images contain **patterns** that are not only due to bio-geochemical processes. These patterns are strongly linked to the **flow dynamics**.

Direct Image Assimilation
Motivations



Sea Surface Temperature



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- Ocean color images contain **patterns** that are not only due to bio-geochemical processes. These patterns are strongly linked to the **flow dynamics**.

- High resolution Ocean color images and SST images usually show very similar submesoscale structures. That is mean that they contain some common information, which is obviously linked with flow dynamics.
- So we may want to exploit these structures to better constrain the dynamic. The key point of direct image assimilation is that we want to be consistent with the considered observed physical model.

Direct Image Assimilation

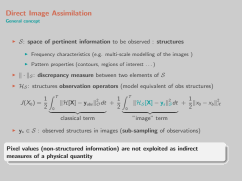
General concept

- $\mathcal{S}$: **space of pertinent information** to be observed : **structures**
  - Frequency characteristics (e.g. multi-scale modelling of the images)
  - Pattern properties (contours, regions of interest . . .)
- $\|\cdot\|_{\mathcal{S}}$: **discrepancy measure** between two elements of $\mathcal{S}$
- $\mathcal{H}_{\mathcal{S}}$: structures **observation operators** (model equivalent of obs structures)

$$J(X_0) = \underbrace{\frac{1}{2}\int_0^T \|\mathcal{H}[\mathbf{X}] - \mathbf{y}_{obs}\|_{\mathcal{O}}^2 dt}_{\text{classical term}} + \underbrace{\frac{1}{2}\int_0^T \|\mathcal{H}_{\mathcal{S}}[\mathbf{X}] - \mathbf{y}_s\|_{\mathcal{S}}^2 dt}_{\text{"image" term}} + \frac{1}{2}\|x_0 - x_b\|_{\mathcal{X}}^2$$

- $\mathbf{y}_s \in \mathcal{S}$: observed structures in images (**sub-sampling** of observations)

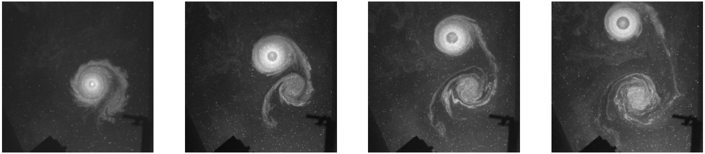
Pixel values (non-structured information) are not exploited as indirect measures of a physical quantity



- *Direct Image Assimilation (DIA)* means that we want to assimilate the image information into the model as it is done with a classical data, *i.e.* by the mean of specific observation operators and norms. *DIA* differs from what I usually call pseudo-observations which pre-process the images to get a data which is represented by the model. This is the case of velocity fields that are inverted from an image sequence and assimilated as an observation of the velocity field. Also DIA differs from classical image sequence analysis techniques because it involves the model of the observed system instead of adding regularization term. In this talk I will also claim that this kind of method may be capable to extract dynamic information from one single ocean tracer image.
- For DIA we need to define what is the pertinent information of the image we want to assimilate. This information should be represented in a mathematical space that can be handled by the assimilation system.
- The norm that computes the discrepancy between two elements in $\mathcal{S}$ should ideally have some good properties for differentiation procedures.
- Finally you need an observation operator that compute the model equivalent of the observed structures. This talk focuses on this last point.
- The cost function of the classical data assimilation system is then augmented with an image part which can be written as follow, where $y^{\mathcal{S}}$ denotes the image data as it is represented in the structure space.

Direct Image (Sequence) Assimilation

Proof of concept with a shallow-water model / turntable experiment



J.-B. Flór and I. Eames, 2002

shallow-water model for $(\mathbf{u},\mathbf{v},\mathbf{h})$

$$(\mathcal{M}) \begin{cases} \partial_t u - u\partial_x u + v\partial_y u - fv + g\partial_x h + \mathcal{D}(u) & = & 0 \\ \partial_t v + u\partial_x v + v\partial_y v + fu + g\partial_y h + \mathcal{D}(v) & = & 0 \\ \partial_t h + \partial_x(hu) + \partial_y(hv) & = & 0 \end{cases}$$

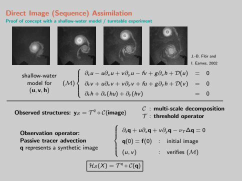
Observed structures: $\mathbf{y}_{\mathcal{S}} = \mathcal{T}^q \circ \mathcal{C}(\textbf{image})$

$\mathcal{C}$: **multi-scale decomposition**
$\mathcal{T}$: **threshold operator**

Observation operator:
Passive tracer advection
$\mathbf{q}$ represents a synthetic image

$$\begin{cases} \partial_t \mathbf{q} + u\partial_x \mathbf{q} + v\partial_y \mathbf{q} - \nu_T \Delta \mathbf{q} = 0 \\ \mathbf{q}(0) = \mathbf{f}(0) \quad : \quad \text{initial image} \\ (u, v) \quad : \quad \text{verifies } (\mathcal{M}) \end{cases}$$

$$\boxed{\mathcal{H}_{\mathcal{S}}(X) = \mathcal{T}^q \circ \mathcal{C}(\mathbf{q})}$$



Framework of the experiment:

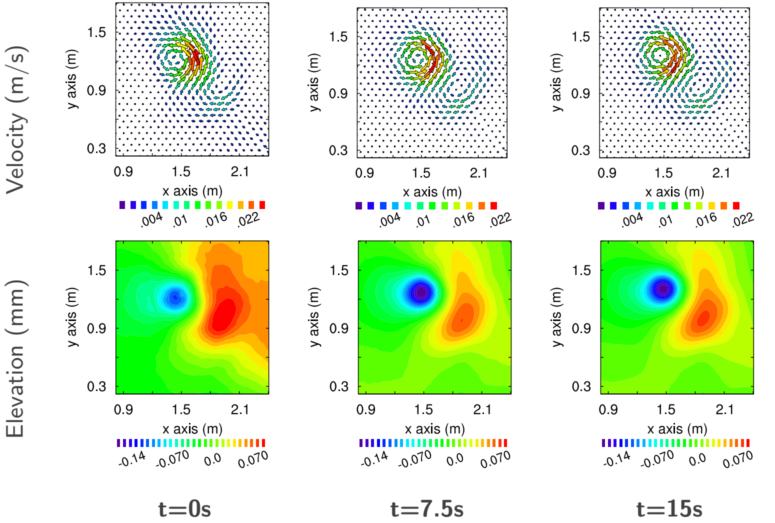
- Observed system (images): fluid flow in a rotating plateform. A vortex is created by stirring and highlighted by a passive tracer (fluorecine).
- This experiment simulates the evolution of a vortex in the atmosphere
- Numerical model of the flow: one-layer shallow-water equations (three state variables: two velocity components and water elevation)

Direct Assimilation of the Image Sequence:

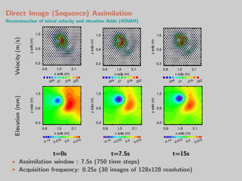
- Image structure space $\mathcal{S}$: subset (threshold) of the curvelet frame
- State variables are not observed
- Observation operator: synthetic image (concentration of a passive tracer, initialised by the first image): the passive tracer concentration is not a state neither a control variable
- Background is the system at rest ($(u, v) = 0, h = h_{mean}$)
- Assimilation scheme : 4D-VAR preconditioned with balance operators (geostrophic balance between h and (u, v)).

Direct Image (Sequence) Assimilation

Reconstruction of initial velocity and elevation fields (4DVAR)



- Assimilation window : 7.5s (750 time steps)
- Acquisition frequency: 0.25s (30 images of 128x128 resolution)

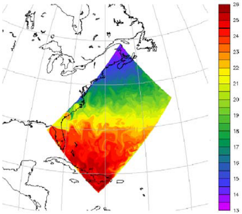


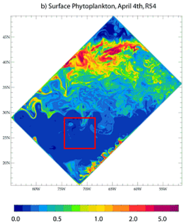
Proof of concept:

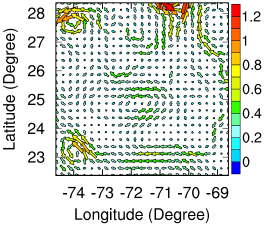
- The vortex is correctly located
- Velocity and elevation fields have a correct structure
- Velocity and elevation fields have correct magnitudes

It is important to notice that this new formalism allows one to get a consistent initial field of the elevation. Classical motion estimation techniques compute a velocity field only.

Test case



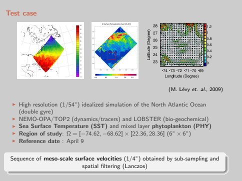




(M. Lévy *et. al.*, 2009)

- High resolution ($1/54^\circ$) idealized simulation of the North Atlantic Ocean (double gyre)
- NEMO-OPA/TOP2 (dynamics/tracers) and LOBSTER (bio-geochemical)
- **Sea Surface Temperature (SST)** and mixed layer **phytoplankton (PHY)**
- **Region of study**: $\Omega = [-74.62, -68.62] \times [22.36, 28.36]$ ($6^\circ \times 6^\circ$)
- **Reference date** : April 9

Sequence of **meso-scale surface velocities** ($1/4^\circ$) obtained by sub-sampling and spatial filtering (Lanczos)



I will now present another way to design observation operators adapted to single ocean tracer images. I will present an impact study that aims to show the relevance of these operators before considering them in a direct assimilation scheme.

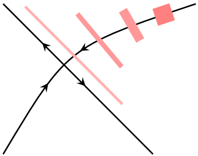
The framework of this experiment is the following:

- I have a one year high resolution simulation of a idealized North Atlantic model in a classical NEMO double-gyre configuration. Dynamics is simulated using NEMO-OPA. We also have a bio-chemical tracers given by the LOBSTER six-compartment model.
- We consider the high resolution Sea Surface Temperature and Mixed-Layer Phytoplankton as our observed images.
- The region of study is located southeast recirculation branch of the Gulf Stream
- As we want to mimic the framework of the assimilation of high resolution images into a meso-scale model we applied a Lanczos filter and a sub-sampling of the velocity field and we consider the surface filtered field as our truth. We can interpret this filtered velocity field as a meso-scale simulation with an ideally parametrized 1/54 degree submesoscale physics.

Coherent Lagrangian Structures (LCS)

The transport of a tracer in a fluid is closely related to emergent patterns called **Coherent Structures** (Ottino 1989, Wiggins 1992):

- Stationary flows: **stable and unstable manifolds of hyperbolic trajectories**
- Delimit regions of **whirls**, **stretching** or **contraction**



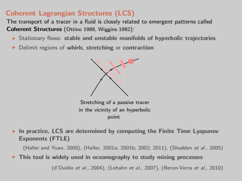
Stretching of a passive tracer in the vicinity of an hyperbolic point

- **In practice, LCS are determined by computing the Finite Time Lyapunov Exponents (FTLE)**

  (Haller and Yuan, 2000), (Haller, 2001a; 2001b; 2002; 2011), (Shadden *et al.*, 2005)

- **This tool is widely used in oceanography to study mixing processes**

  (d'Ovidio *et al.*, 2004), (Lehahn *et al.*, 2007), (Beron-Verra *et al.*, 2010)



- For a stationary flow LCS correspond to stable and unstable manifolds of hyperbolic trajectories.
- Generalizing this concept for non stationary flows was not obvious and still few rigorous work exists
- It is now admitted that LCS are maximizing the ridges of FLTE field
- "In practice" means "when the velocity field is only known as a *finite* data set."

Finite-Time Lyapunov Exponents and Vectors (FTLE & FTLV)

FTLE represents the **rate of separation** of initially neighboring particles over a **finite-time window** $[0, T]$

$$(\star)\begin{cases} \dfrac{D\mathbf{x}(t)}{Dt} = \mathbf{u}(\mathbf{x}(t), t) \\ \mathbf{x}(t_0) = \mathbf{x}_0 \end{cases} \qquad \begin{cases} \dfrac{D\delta\mathbf{x}(t)}{Dt} = \nabla\mathbf{u}(\mathbf{x}(t), t).\delta\mathbf{x}(t) \\ \delta\mathbf{x}(t_0) = \delta_0, \quad \mathbf{x}(t_0) = \mathbf{x}_0 \end{cases}$$

Particle transport by the flow $\mathbf{u}(\mathbf{x}, t)$

Evolution of a given perturbation $\delta\mathbf{x}$

Cauchy-Green strain tensor

$$\Delta = \left[\nabla\phi_{t_0}^{t_0+T}(\mathbf{x}_0)\right]^* \left[\nabla\phi_{t_0}^{t_0+T}(\mathbf{x}_0)\right], \quad \phi_{t_0}^{t_0+T} : \mathbf{x_0} \mapsto \mathbf{x}(T), \quad \text{flow map of } (\star)$$

Maximum stretching occurs when $\delta\mathbf{x}(0)$ is aligned with the eigenvector associated to the **largest eigenvalue** $\lambda_{\max}$ of Δ

- **Finite-Time Lyapunov Vector** : eigenvector $\varphi_{t_0}^{t_0+T}(\mathbf{x}_0)$ associated to $\lambda_{\max}$
- **Finite-Time Lyapunov Exponent** :

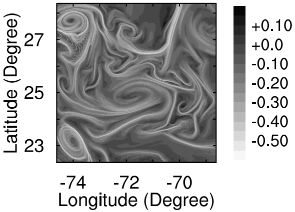
$$\sigma_{t_0}^{t_0+T}(\mathbf{x}_0) = \frac{1}{|T|} \ln \sqrt{\lambda_{\max}(\Delta)}$$

- **Backward FTLE&V** (stable manifold): time integration is inverted in $(\star)$

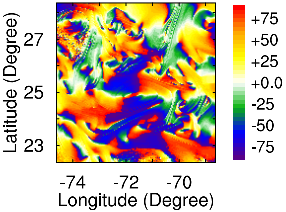
(Ott, 1993), (Shadden *et al.* ,2005; 2009), (Haller, 2011)

FTLE and FTLV: variational point of view

- **FTLE and FTLV are local notions**: the scalar $\sigma_{t_0}^{t_0+T}$ and the eigenvector $\varphi_{t_0}^{t_0+T}$ are computed at a given point $\mathbf{x}_0$
- Seeding a domain with particles initially located on a grid leads to the computation of a discretized scalar (FTLE) and vector (FTLV) fields
- **Ridges of backward FTLE field approximate LCS** (Haller, 2011).



FTLE (day^{-1})



FTLV orientations (Degree)

Backward integration
Meso-scale velocity field
Resolution 1/54°

FTLE and FTLV orientation maps with respect to the velocity field $\mathbf{u}$

$$\Sigma[\mathbf{u}] \;:\; \mathbf{x} \in \Omega \rightarrow \sigma_{t_0}^{t_0+T}(\mathbf{x}) \in \mathbb{R} \quad \text{and} \quad \Phi[\mathbf{u}] \;:\; \mathbf{x} \in \Omega \rightarrow \varphi_{t_0}^{t_0+T}(\mathbf{x}) \in \mathbb{R}^2$$

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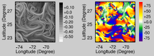


FTLE (day^{-1}) FTLV orientations (Degree)

Backward integration
Meso-scale velocity field
Resolution 1/54°

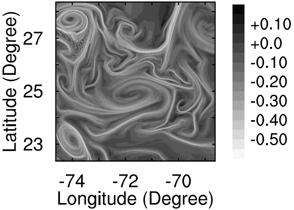
FTLE and FTLV orientation maps with respect to the velocity field u

$$\Sigma[\mathbf{u}] \ : \ \mathbf{x} \in \Omega \rightarrow \sigma_{t_0}^{t_0+T}(\mathbf{x}) \in \mathbb{R} \quad \text{and} \quad \Phi[\mathbf{u}] \ : \ \mathbf{x} \in \Omega \rightarrow \varphi_{t_0}^{t_0+T}(\mathbf{x}) \in \mathbb{R}^2$$

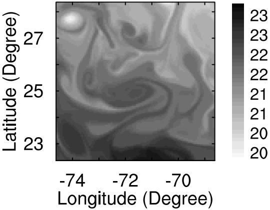
- Our study focuses on the sensitivity of the FTLE and FTLV orientation distribution to perturbations on the velocity field.
- For that we adopt a variational approach by considering the operators Σ and Φ that maps the **meso-scale** velocity field onto the FTLE and FTLV orientation distribution.
- We suppose that the time advection T is fixed (i.e. imposed by the assimilation scheme)

Connection between FTLE and tracer fields

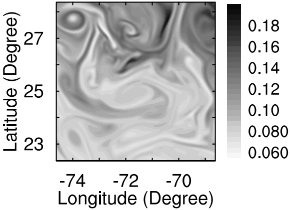
High resolution backward FTLE fields computed from a **meso-scale** $(1/4^\circ)$ velocity field show contours that correspond reasonably well to the main **submesoscale** $(1/54^\circ)$ patterns of the tracer filed at the reference date



$1/54^\circ$ FTLE



$1/54^\circ$ SST



$1/54^\circ$ PHY

(Beron-Vera *et al.*, 2010; Olascoaga *et al.*, 2006;2008)
(Shadden *et al.*, 2009) (Y. Lehahn *et. al.*, 2007)
(F. d'Ovidio *et. al.* 2004, 2009)

Observation operator based on FTLE

- **Structure space** $\mathcal{E} = \{c \in \mathcal{I}_\Omega \;:\; \Omega \;\rightarrow\; \{0,1\}\}$ **(binary images)**
- **Contour extraction (gradient threshold)**

$$\mathrm{E} \;:\; \mathcal{I}_\Omega \rightarrow \mathcal{E} \qquad \mathrm{E}(c)(i,j) = \begin{cases} 1 & \text{if} \quad \|\nabla c(i,j)\| > \epsilon \\ 0 & \text{else} \end{cases}$$

- **Discrepancy** between c and $c^\star$ in $\mathcal{E}$

$$\|c - c^\star\|_{\mathcal{E}} = \sqrt{\frac{1}{n \times m} \sum_{i,j} |c(i,j) - c^\star(i,j)|^2}$$

- **Velocity field sequence in the window [-T,0]**: $\mathbf{u} = (\mathbf{u}_k)_{k=-T}^{k=0}$
- **Observation operator**

$$\mathcal{H}_{\mathcal{E}}(X) = \mathrm{E}(\Sigma(\mathbf{u})) \qquad \Sigma(\mathbf{u}) \;:\; \mathbf{x} \in \Omega \mapsto {\sigma_0}^{-T}(\mathbf{x}) \in \mathbb{R}$$

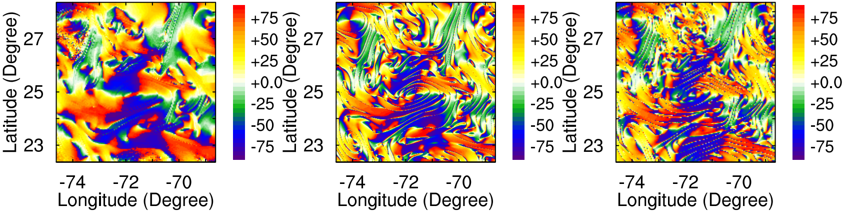
Cost function associated to the triplet $\mathrm{E} = (\mathcal{H}_{\mathcal{E}}, \mathcal{E}, \|\cdot\|_{\mathcal{E}})$

$$J_{\mathcal{E}}(\mathbf{u}) = \|\mathrm{E}_{\epsilon'}(\Sigma(\mathbf{u})) - \mathrm{E}_{\epsilon}(c)\|_{\mathcal{E}}^2$$

Connection between FTLV and tracer fields

The orientation of the gradient of passive tracers converge to that of backward FTLV in freely decaying 2D turbulence flow

(Lapeyre, 2002)



FTLV orientations   ∇ SST orientations   ∇ PHY orientations

This property has also been observed on real data

(d'Ovidio *et al.*, 2009)

Observation Operator based on FTLV

- **Structure Space**: functions with values in the Euclidean unit sphere S^2

$$\mathcal{V} = \{f : \Omega \to S^2\}$$

- Orientation of $\mathbf{v} = (u, v) \in S^2$: $\Theta(\mathbf{v}) = \mathrm{atan}(v) \in [-\pi/2, \pi/2]$
- **Angular measure** in $\mathcal{V}$

$$\|f - g\|_{\mathcal{V}} = \sqrt{\frac{1}{n \times m} \sum_{i,j} \sin^2[\Theta(f(i,j)) - \Theta(g(i,j))]}$$

- **Observation Operator**

$$\mathcal{H}_{\mathcal{V}}(X) = \Phi(\mathbf{u}) \qquad \Phi(\mathbf{u}) \;:\; \mathbf{x} \in \Omega \mapsto \varphi_0^{-T}(\mathbf{x}) \in S^2$$

- **Information extraction** from the observed image c

$$\mathrm{V} \;:\; \mathcal{I}_{\Omega} \to \mathcal{V} \qquad \mathrm{V}(c)(i,j) = \frac{\nabla c(i,j)}{\|\nabla c(i,j)\|} = \mathbf{y}^s$$

Cost function associated to the triplet $\mathrm{V} = (\mathcal{H}_{\mathcal{V}}, \mathcal{V}, \|\cdot\|_{\mathcal{V}})$

$$J_{\mathcal{V}}(\mathbf{u}) = \|\Phi(\mathbf{u}) - \mathrm{V}(c)\|^2_{\mathcal{V}}.$$

Methodology

Pre-requisite for data assimilation

Aim: study the behaviour of the cost function with respect to the **amplitude** λ **of velocity perturbations** on the form $\mathbf{u}_0 + \lambda\delta\mathbf{u}$

Sequence of perturbed velocity fields

$$\mathbf{u}_k^\lambda = \begin{cases} \mathbf{u}_0 + \lambda\delta\mathbf{u} & \text{if} \quad k = 0 \\ \mathbf{u}_k & \text{else} \end{cases} \qquad \mathbf{u}^\lambda = (\mathbf{u}_k^\lambda)_{k=-10}^{k=0}$$

Sensitivity of the cost function w.r.t. the data $\mathbf{y}^\mathcal{S}$

$$\tilde{J}_\mathcal{S}(\lambda) = \|\mathcal{H}_\mathcal{S}[\mathbf{u}^\lambda] - \mathbf{y}^\mathcal{S}\|_\mathcal{S}^2, \quad \lambda \in \Lambda.$$

Before exploiting the triplet $(\mathcal{H}_\mathcal{S}, \mathcal{S}, \|\cdot\|_\mathcal{S})$ in an assimilation scheme it is necessary to check that the **sensitivity function** $\tilde{\mathcal{J}}_\mathcal{S}$ **admits a minimum at** $\lambda = 0$ (**no perturbation**).

Methodology

Climatological model for the velocity field sequence

- $(\mathbf{u}^{(l)})_{l=1}^{r}$: first $r = 100$ EOFs of the one year sequence of simulated surface velocity fields

$$\mathbf{u}_k = \overline{\mathbf{u}} + \sum_{l=1}^{m=209} \alpha_k^{(l)} \mathbf{u}^{(l)},$$

- $\mathbf{S} = (\mathbf{u}^{(1)}|\mathbf{u}^{(2)}| \cdots |\mathbf{u}^{(r)})$: reduced rank square root representation of the climatological covariance matrix

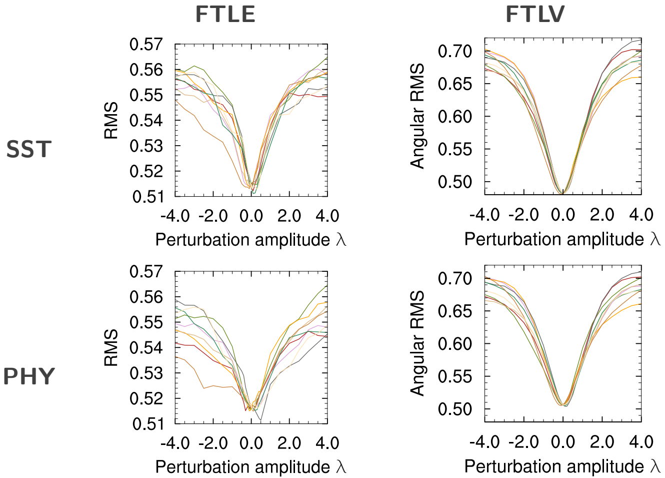
$$\mathbf{P} = \frac{1}{m} \sum_{k=1}^{m+1} (\mathbf{u}_k - \overline{\mathbf{u}})(\mathbf{u}_k - \overline{\mathbf{u}})^*$$

- **Gaussian perturbations with zero mean and covariance SS***

$$\delta_{\mathbf{u}} \sim \mathcal{N}(0, \mathbf{S}\mathbf{S}^{\mathsf{T}}). \quad \delta_{\mathbf{u}} = \sum_{l=1}^{r} \mathbf{u}^{(l)} \delta x_l \quad \text{with} \quad \delta x_l \sim \mathcal{N}(0, 1)$$

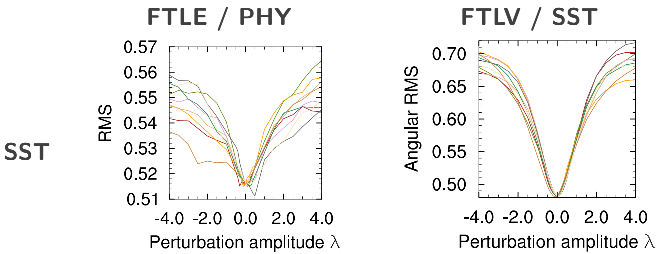
We are interested in perturbations of amplitude λ applied at the **reference date**:
$\mathbf{u}_0 + \lambda \delta \mathbf{u}$

Results

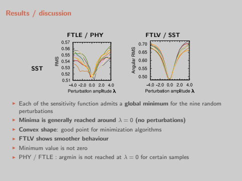


Variation of the sensitivity functions based on FTLE and FTLV
Variation are computed w.r.t. the amplitude λ of nine random perturbations
SST and PHY data

Results / discussion



- Each of the sensitivity function admits a **global minimum** for the nine random perturbations
- **Minima is generally reached around $\lambda = 0$ (no perturbations)**
- **Convex shape**: good point for minimization algorithms
- **FTLV shows smoother behaviour**
- Minimum value is not zero
- PHY / FTLE : argmin is not reached at $\lambda = 0$ for certain samples



- *Minimum values are not zero*: This is not surprising because the Lagrangian tool is known to provide only an incomplete representation of the SST and MLP dynamics. Note, however, that for our application, this is not unsatisfactory. Several reasons can be put forward to explain that: The main reason is probably because ocean tracers such as SST and MLP have their own dynamics that cannot be observed by the Lagrangian tool. The high-resolution tracer gradients also depend on submesoscale dynamics; these dynamics are not taken into account in the computation of FTLE-V because they are computed from a mesoscale field. In addition, FTLE-Vs have been computed at the ocean surface and we know that patterns in ocean colour images (MLP field) are a surface signature of a three-dimensional process. The underlying dynamics also intervene in the formation of these patterns.
- *Some realisations of the sensitivity functions do not reach their minimum at zero:* This is particularly the case for the FTLE-based triplet, the worse being with MLP data. We also observe the same problem with this tracer for the FTLV-based triplet, but it is less marked. Such behaviour reveals that the data assimilation problem is not well-posed in the Hadamard sense, a situation quite common with such inverse problems. Regularization is needed.

Conclusions, future work and references

Conclusions

- **High resolution** ocean tracer images may be exploited by a direct image assimilation scheme in a **mesoscale** model
- **FTLE and FTLV fields contain information about the system dynamic** that can be observed in the ocean tracer fields: they are good candidates to construct **observation operators** for image assimilation
- A single ocean tracer image contains a **time integrated information** on the system dynamics

Future work

- Full data assimilation experiment
- Observation errors

References

- O. Titaud, J.-M. Brankart, J. Verron, On the use of Finite-Time Lyapunov Exponents and Vectors for direct assimilation of tracer images into ocean models, *Tellus A*, *in press*
- O. Titaud, A. Vidard, I. Souopgui, and F.-X. Le Dimet. Assimilation of image sequences in numerical models. Tellus A, 62(1):30-47, Janvier 2010