Accelerating minimizations in ensemble variational assimilation

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Outline

1. Ensemble Variational assimilation
2. Accelerating minimizations
3. Conclusion and future work

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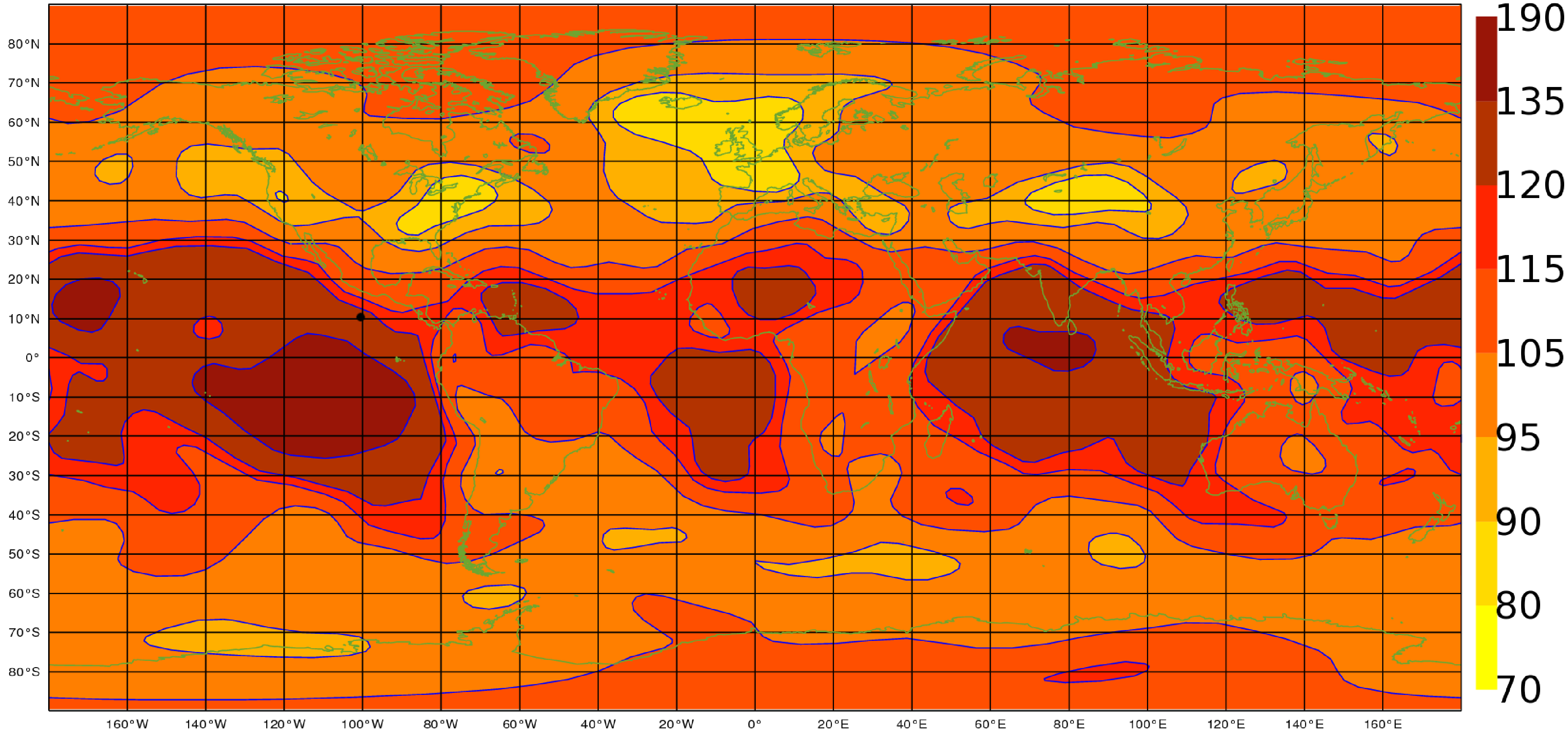
The operational Météo-France ensemble Var assimilation

- 1st operational implementation of En Var assim. (2008; ECMWF, 2010-11).
- Six perturbed members, T399 L70 (50 km), with global 4D-Var Arpege.
- Spatial filtering of error variances.
- Inflation of ensemble **B** / model error contributions.
- Flow-dependent background error variances in 4D-Var (and EnDA), for minimizations (all variables) and observation QC.
- Initialization of Météo-France ensemble prediction by EnDA.
- Flow-dependent background error correlations experimented.

Background error correlations using EnDA and wavelets



Wavelet-implied horizontal length-scales (in km),
for wind near 500 hPa, averaged over a 4-day period.

(Varella et al 2011b, and also Fisher 2003,
Deckmyn and Berre 2005, Pannekoucke et al 2007)

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Ensemble variational assimilation

- Minimize N cost-functions J_n, n=1, N, with perturbed innovations $\mathbf{d}_n$:

$$J_n(\delta \mathbf{x}_n) = 1/2\ \delta \mathbf{x}_n^T\ \mathbf{B}^{-1}\ \delta \mathbf{x}_n + 1/2\ (\mathbf{d}_n - \mathbf{H}_n\ \delta \mathbf{x}_n)^T\ \mathbf{R}^{-1}\ (\mathbf{d}_n - \mathbf{H}_n\ \delta \mathbf{x}_n),$$

with $\mathbf{B}$ and $\mathbf{R}$ background and observation error matrices,
and $\mathbf{d}_n = \mathbf{y}^o + \mathbf{R}^{1/2}\eta^o_n - H(M(\mathbf{x}^b_n))$, with η^o_n a vector of random numbers.

$$\mathbf{x}^a_n = \mathbf{x}^b_n + \delta \mathbf{x}_n$$

- Perturbed backgrounds for the next analyses:

$\mathbf{x}^b{}_n^+ = M(\mathbf{x}^a_n) + \mathbf{Q}^{1/2}\eta^m_n$, with η^m_n a vector of random numbers and $\mathbf{Q}$ model error covariance matrix.

Hessian matrix of the assimilation problem

- Hessian of the cost-function:

$$J'' = \mathbf{B}^{-1} + \mathbf{H}^{T} \mathbf{R}^{-1} \mathbf{H}.$$

- Bad conditioning of J'': very slow (or no) convergence.

- Cost-function with $\mathbf{B}^{1/2}$ preconditioning ($\delta\mathbf{x} = \mathbf{B}^{1/2}\chi$):

$$J(\chi) = 1/2 \; \chi^{T}\chi + 1/2 \; (\mathbf{d}-\mathbf{H}\;\mathbf{B}^{1/2}\chi)^{T} \; \mathbf{R}^{-1} \; (\mathbf{d}-\mathbf{H}\;\mathbf{B}^{1/2}\chi).$$

- Hessian of the cost-function:

$$J'' = \mathbf{I} + \mathbf{B}^{T/2}\mathbf{H}^{T}\mathbf{R}^{-1}\;\mathbf{H}\;\mathbf{B}^{1/2}.$$

- Far better conditioning and convergence!
(Lorenc 1988, Haben et al 2011)

Lanczos algorithm

- Generate iteratively a set of K orthonormal vectors **q** such as

$$\mathbf{Q}_K^T \, \mathbf{J}'' \, \mathbf{Q}_K = \mathbf{T}_k,$$

where $\mathbf{Q}_K = (\mathbf{q}_1 \, \mathbf{q}_2 \ldots \mathbf{q}_K)$, and $\mathbf{T}_K$ is a tri-diagonal matrix.

- The extremal eigenvalues of $\mathbf{T}_K$ quickly converge towards the extremal eigenvalues of **J**''.

- If $\mathbf{T}_K = \mathbf{Y}_K \Lambda_K \mathbf{Y}_K^T$ is the eigendecomposition of $\mathbf{T}_K$, the Ritz vectors are obtained with

$$\mathbf{Z}_K = \mathbf{Q}_K \mathbf{Y}_k$$

and the Ritz pairs $(\mathbf{z}_k, \lambda_k)$ approximate the eigenpairs of **J**''.

Lanczos algorithm / Conjugate gradient

- Use of the Lanczos vectors to get the solution of the variational problem:

$$\chi_K = \chi_0 + \mathbf{Q}_K \Omega_K .$$

- Optimal coefficients Ω_k should make the gradient of J vanish at χ_K:

$$\begin{aligned}\mathbf{J}'(\chi_K) &= \mathbf{J}'(\chi_0) + \mathbf{J}'' (\chi_K - \chi_0) \\ &= \mathbf{J}'(\chi_0) + \mathbf{J}'' \mathbf{Q}_K \Omega_K \\ &= \mathbf{0},\end{aligned}$$

which gives

$$\begin{aligned}\Omega_K &= - (\mathbf{Q}_K^T \mathbf{J}'' \mathbf{Q}_K)^{-1} \mathbf{Q}_K^T \mathbf{J}'(\chi_0) \\ &= - \mathbf{T}_K^{-1} \mathbf{Q}_K^T \mathbf{J}'(\chi_0),\end{aligned}$$

and then

$$\chi_K = \chi_0 - \mathbf{Q}_K \mathbf{T}_K^{-1} \mathbf{Q}_K^T \mathbf{J}'(\chi_0).$$

- Same solution as after K iterations of a Conjugate Gradient algorithm. (Paige and Saunders 1975, Fisher 1998)

Accelerating a « perturbed » minimization using « unperturbed » Lanczos vectors

- Minimizations with
  - unperturbed innovations **d** and
  - perturbed innovations $\mathbf{d}_n$ have basically the same Hessians:

$$J''(\mathbf{d}) = \mathbf{I} + \mathbf{B}^{T/2}\mathbf{H}^{T}\mathbf{R}^{-1}\,\mathbf{H}\,\mathbf{B}^{1/2},$$
$$J''(\mathbf{d}_n) = \mathbf{I} + \mathbf{B}^{T/2}\mathbf{H}_n^{T}\mathbf{R}^{-1}\,\mathbf{H}_n\,\mathbf{B}^{1/2},$$

- The solution obtained for the « unperturbed » problem

$$\chi_K = \chi_0 - \mathbf{Q}_K\,(\,\mathbf{Q}_K^{T}\,J''\,\mathbf{Q}_K\,)^{-1}\,\mathbf{Q}_K^{T}\,\mathbf{J}'(\chi_0, \mathbf{d})$$

can be transposed to the « perturbed » minimization

$$\chi_{K,n} = \chi_0 - \mathbf{Q}_K\,(\,\mathbf{Q}_K^{T}\,J''\,\mathbf{Q}_K\,)^{-1}\,\mathbf{Q}_K^{T}\,\mathbf{J}'(\chi_0, \mathbf{d}_n)$$

to improve its starting point.

Accelerating minimizations using « perturbed » Lanczos vectors

- If N perturbed minimizations, with K iterations, already performed, then the starting pt of a perturbed (or unpert.) minim. can be written

$$\chi_K = \chi_0 + \mathbf{Q}_{K,N}\,\Omega_{K,N},$$

  where $\Omega_{K,N}$ is a vector of N x K coefficients and

$$\mathbf{Q}_{K,N} = (\mathbf{q}_{1,1} \cdots \mathbf{q}_{K,1} \cdots \mathbf{q}_{1,N} \cdots \mathbf{q}_{K,N})$$

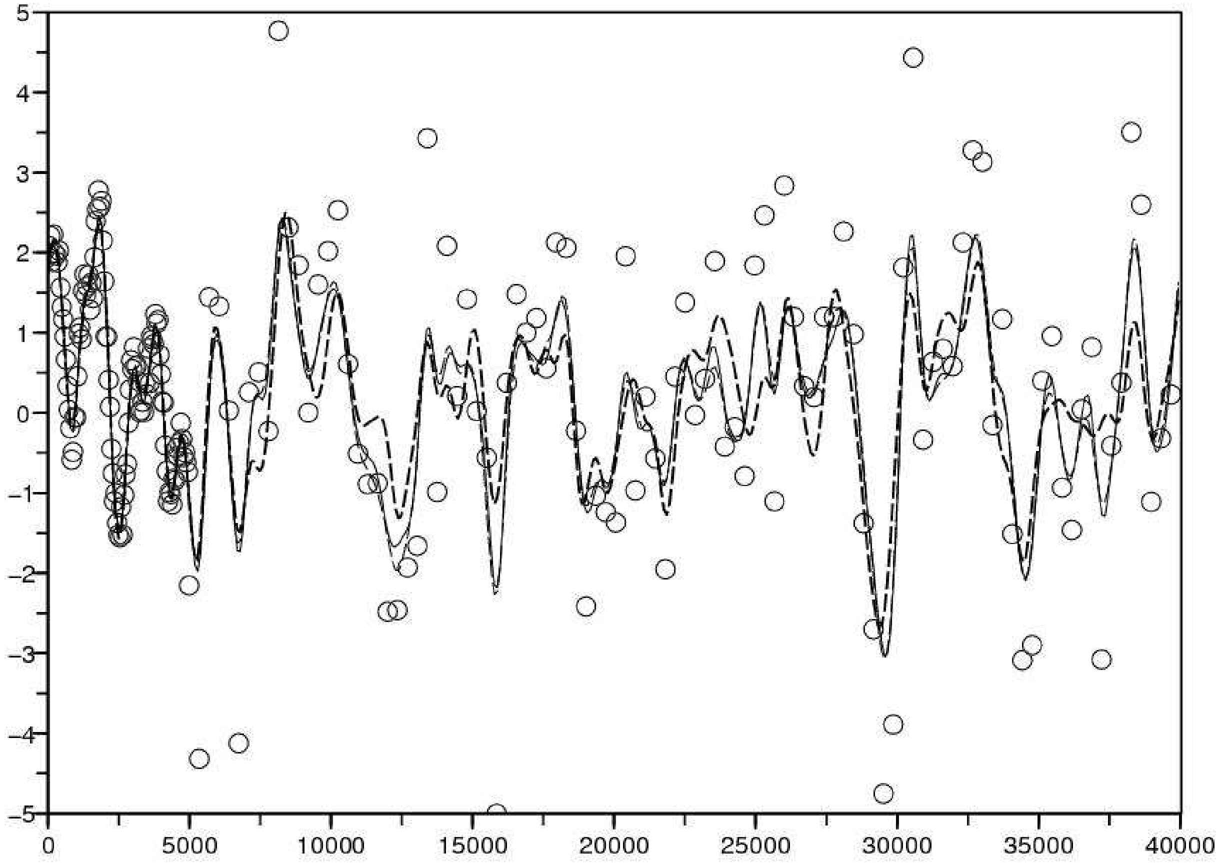
  is a matrix containing the N x K Lanczos vectors.

- Following the same approach as above, the solution can be expressed:

$$\chi_{K,N} = \chi_0 - \mathbf{Q}_{K,N}\,(\mathbf{Q}_{K,N}^T\,\mathbf{J}''\,\mathbf{Q}_{K,N})^{-1}\,\mathbf{Q}_{K,N}^T\,\mathbf{J}'(\chi_0).$$

- Matrix $\mathbf{Q}_{K,N}^T\,\mathbf{J}''\,\mathbf{Q}_{K,N}$ is no longer tri-diagonal, but can be easily inverted.

Accelerating minimizations using N sets of « perturbed » Lanczos vectors (K = 10)



n=401 (δs = 100km)

p=200 (δs^o=50/350km)

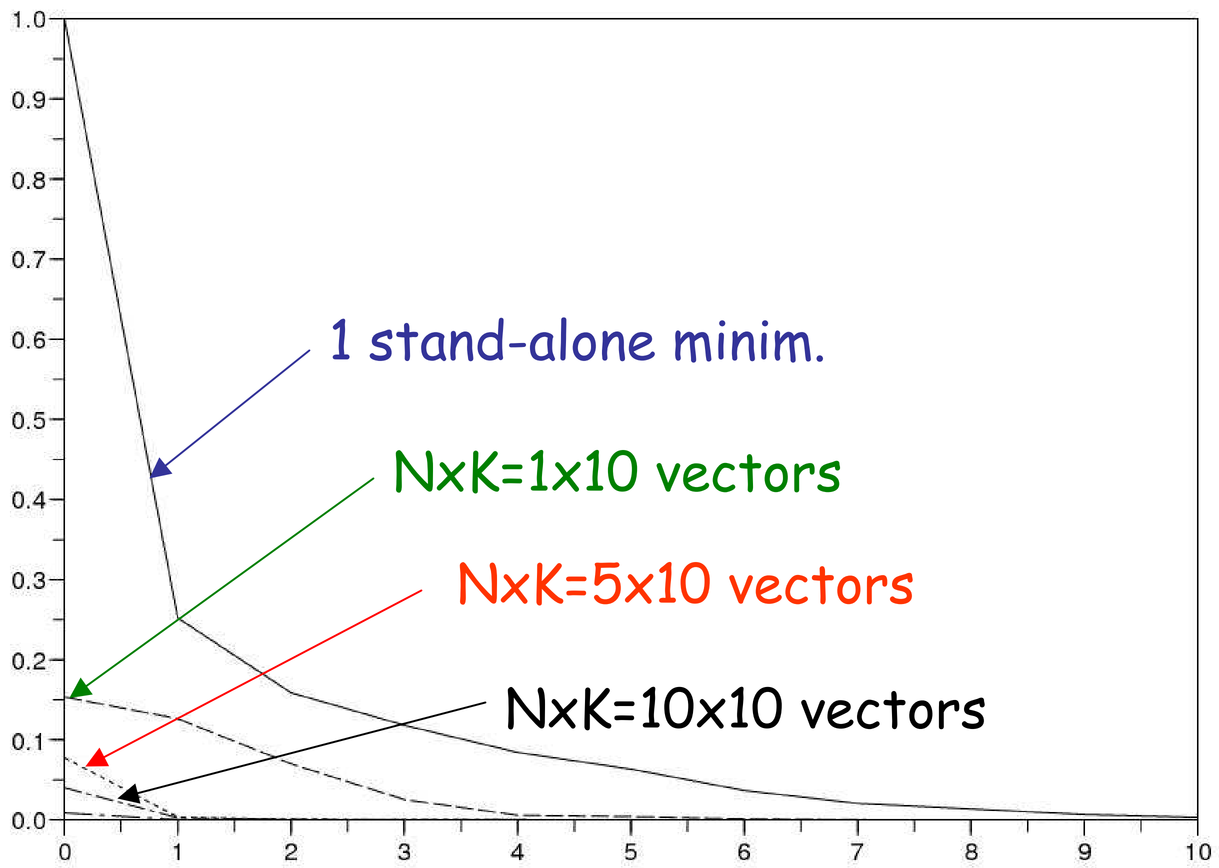
σ^b=1, L^b=300km

σ^b=0,33/1, L^o=0km

Thin solid line: exact perturbed analysis

Thick dashed line : starting point with NxK = 10x10 vectors

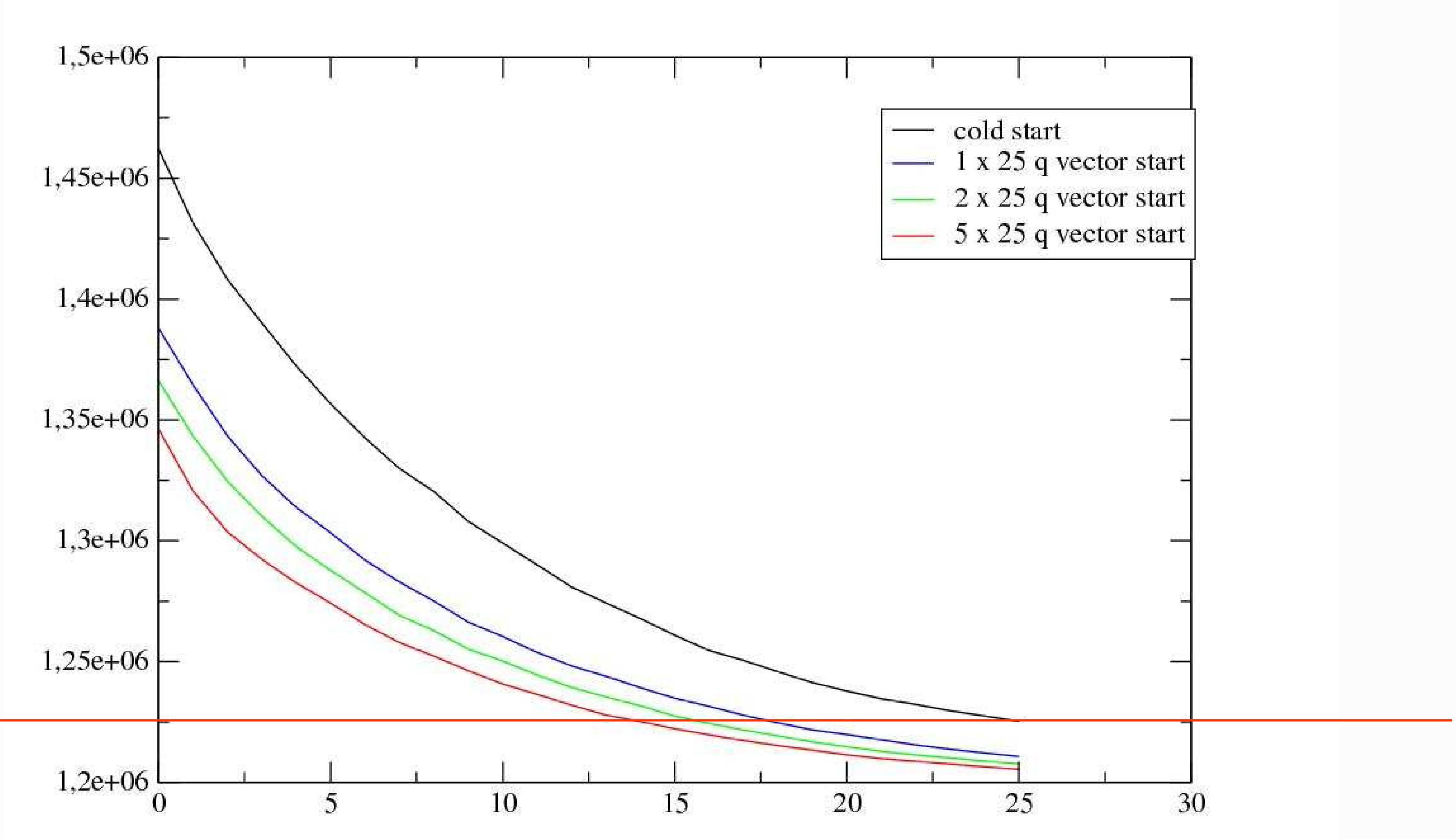
Accelerating minimizations using N sets of « perturbed » Lanczos vectors (K=10)



Starting point and decrease of the cost function

for a new « perturbed » toy minimization

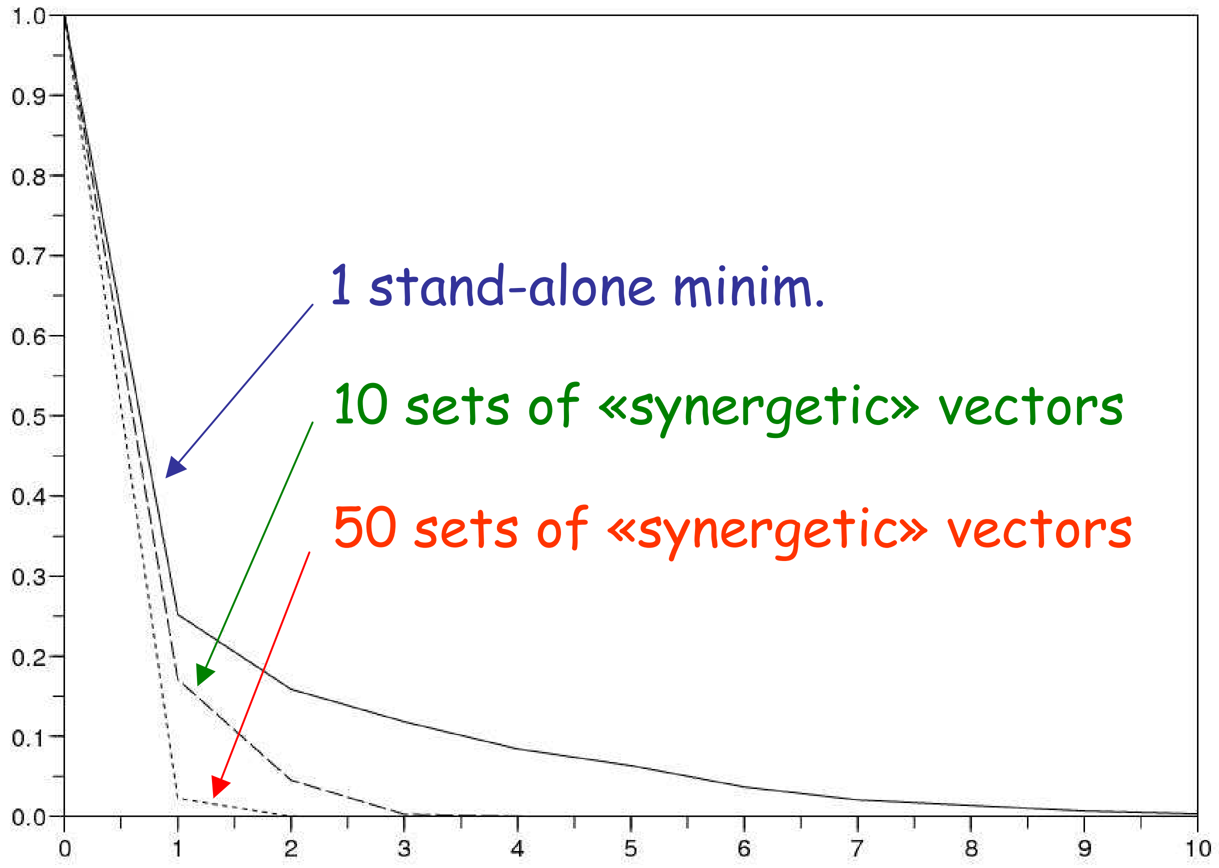
Real size application : use of N sets of « perturbed » 4D-Var Lanczos vectors (K = 25)



Starting point and decrease of the cost function

for a new « perturbed » 4D-Var Arpege minimization

Block Lanczos minimizations using « perturbed » Lanczos vectors



Decrease of the cost function
for a particular « perturbed » toy minimization

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Conclusion and future work

- Ensemble Variational assimilation:
  error cycling can be simulated in a way consistent with 4D-Var.

- Flow-dependent covariances can be estimated.

- Accelerating minimizations seems possible
  (preliminary tests in real size 4D-Var EnDA Arpege also encouraging).

- Connection with Block Lanczos / CG algorithms (O'Leary 1980).

- Possible application in EnVar without TL/AD
  (Lorenc 2003, Buehner 2005).