Validity of hydrostatic balance in the perturbations generated by the Met Office high resolution ensemble forecast system

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May 20, 2009

Funding: NERC & Met Office

Motivation

Data assimilation algorithms:

$$x^a = x^f + K(y - Hx^f)$$

$$K = PH^T(HPH^T + R)^{-1}$$

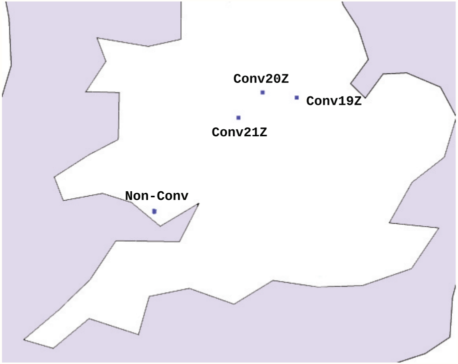
- Initialization of P is very important especially for variational methods
- Incorrect specification of forecast errors can lead to a degraded forecast

The questions:

- Does hydrostatic balance holds in forecast perturbations at high resolutions?
- If it does then to what extent?

Case study of 3h window on 27/07/2008 over southern UK:

- Model used is Met Office Global and Regional Ensemble Prediction System (MOGREPS)
- Model has 1.5km horizontal resolution
- MOGREPS uses ETKF methodology to determine a set of perturbed forecasts and 1.5km UM forward model
- Model is initialized with a reconfigured ensemble from 24km to 1.5km at 18Z
- 24 ensemble member forecasts available at 19Z, 20Z, 21Z
- Domain size $360 \times 288 \times 70$



Available fields for all vertical levels are

- Exner pressure Π
- potential temperature θ
- and specific humidity q

Let the vertical state vector be defined as

$$\mathbf{x} = (\Pi_{0,\ldots,k-1}, \theta_{0,\ldots,k-1}, q_{0,\ldots,k-1}),$$

where $k = 70$ is number of vertical levels and an ensemble is defined as

$$\mathbf{X} = [\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{N-1}] \in \mathcal{R}^{3k \times N}$$

where $N = 24$ is the number of ensemble members.

The ensemble mean or control is defined as $\bar{\mathbf{x}}$. Thus, the ensemble perturbations are given by

$$\mathbf{X}'_i = \mathbf{X}_i - \bar{\mathbf{x}}$$

where $i = 0, \ldots, N-1$.

Hydrostatic balance is given by (e.g. see Wallace 2006),

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

where $p(z)$ is pressure, $T(z)$ is temperature, z is vertical height, and $R = 287.06$ J K^{-1}kg^{-1} is the gas constant for dry air.
Using expression for Exner pressure $\Pi = \left(\frac{p}{p_0}\right)^{R/c_p} = \frac{T}{\theta^v}$ and virtual potential temperature $\theta^v = \theta\left(1 + (\epsilon^{-1} - 1)q\right)$, hydrostatic equation above can be written as follows

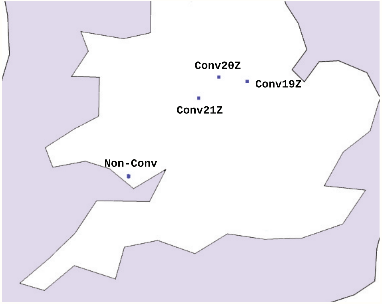
$$\frac{d\Pi}{dz} = -\frac{g}{c_p}\left(1 + (\epsilon^{-1} - 1)q\right)^{-1}\theta^{-1}.$$

By decomposing $\Pi = \overline{\Pi} + \Pi'$, $\theta = \overline{\theta} + \theta'$ and $q = \overline{q} + q'$ above equation may be linearised, giving a first order approximation to hydrostatic equation for the perturbations:

$$\frac{d\Pi'}{dz} = \frac{g}{c_p}\left[\frac{(\epsilon^{-1}-1)q'}{(1+(\epsilon^{-1}-1)\overline{q})^2\overline{\theta}} + \frac{\theta'}{(1+(\epsilon^{-1}-1)\overline{q})\overline{\theta}^2}\right].$$

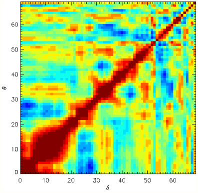
Rearranging gives

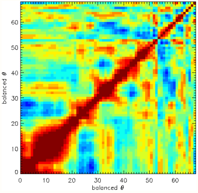
$$\theta'_H = -\frac{(\epsilon^{-1}-1)q'\overline{\theta}}{1+(\epsilon^{-1}-1)\overline{q}} + \frac{c_p}{g}\frac{d\Pi'}{dz}\overline{\theta}^2(1+(\epsilon^{-1}-1)\overline{q}).$$



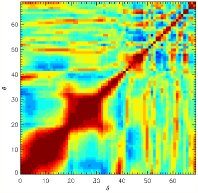
- Compute θ'_H for each column
- Compare correlation matrices of θ' and θ'_H
- Examine explained variances and error flow
- Aggregate data around Conv19Z and NonConv points to examine hydrostatic balance as a function of horizontal scale

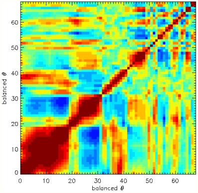
Correlation matrices for 19Z at 1.5km resolution





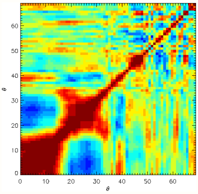
(a) θ', Non-Conv (b) θ'_H, Non-Conv



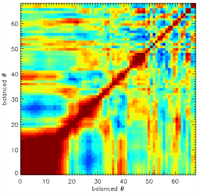


(c) θ', Conv19Z (d) θ'_H, Conv19Z

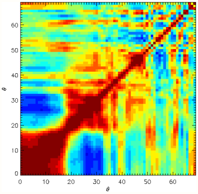
Correlation matrices for 19Z at 4km and 12km resolution



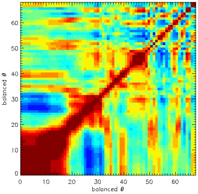
(a) θ', Conv4km



(b) θ'_H, Conv4km



(c) θ', Conv12km



(d) θ'_H, Conv12km

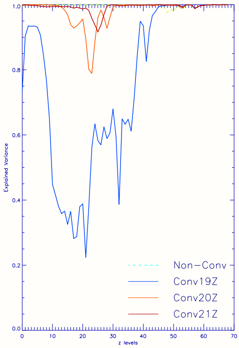
Balance measure

The explained variance is given by

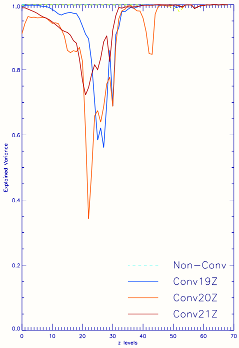
$$E(z) = \left(1 - \frac{\sigma_U^2(z)}{\sigma^2(z)}\right) \tag{1}$$

where σ^2 is the grid-point variance of θ' and σ_U^2 is the variance of the unbalanced part of the perturbations, i.e. $\theta'_U = \theta' - \theta'_H$, and z is the vertical level. Thus, if $E \approx 1$ then perturbations are close to hydrostatic balance and if $E \approx 0$ then they are imbalanced.

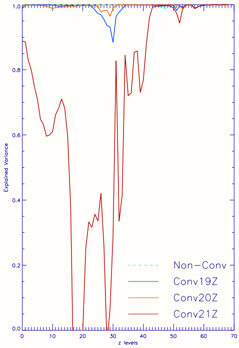
Explained variance at 1.5km resolution



(e) E(z) at 19Z

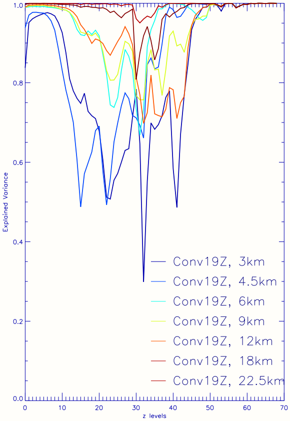


(f) E(z) at 20Z



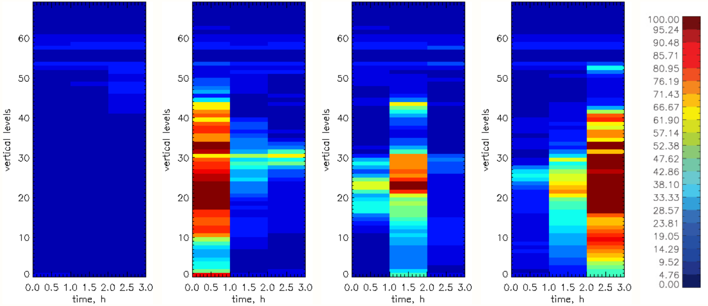
(g) E(z) at 21Z

Explained variance at coarser resolution



(h) E(z) at 19Z

$$rel.error = \frac{\sqrt{\overline{(\theta'_H - \theta')^2}}}{\overline{|\theta'_H|}} \times 100$$



(a) Non-Conv (b) Conv19Z (c) Conv20Z (d) Conv21Z

- At 1.5km resolution hydrostatic balance does not hold in the perturbations in the regions of convection but it does hold in the regions where convection is not present.

- This suggests that hydrostatic balance should be relaxed around convective columns and levels in the correlation matrices at 1.5km resolution. A way of achieving this would be by redesigning the control variable transform in UM.

- $\approx$ 20km horizontal resolution is the limit at which the hydrostatic balance becomes valid over the entire domain.

Future work

- Investigate balance properties of P using an idealised 1+2D convective model and EnSRF
- Further, how balances are affected by applying localization
- Test performance of EnSRF when applied to a convective model

Thank You!