Inverse Shallow Water Flow Modeling using Model Reduction

Muhammad Umer Altaf, Arnold Heemink, Martin Verlaan
Delft Institute of Applied Mathematics
TUDelft
The Netherlands

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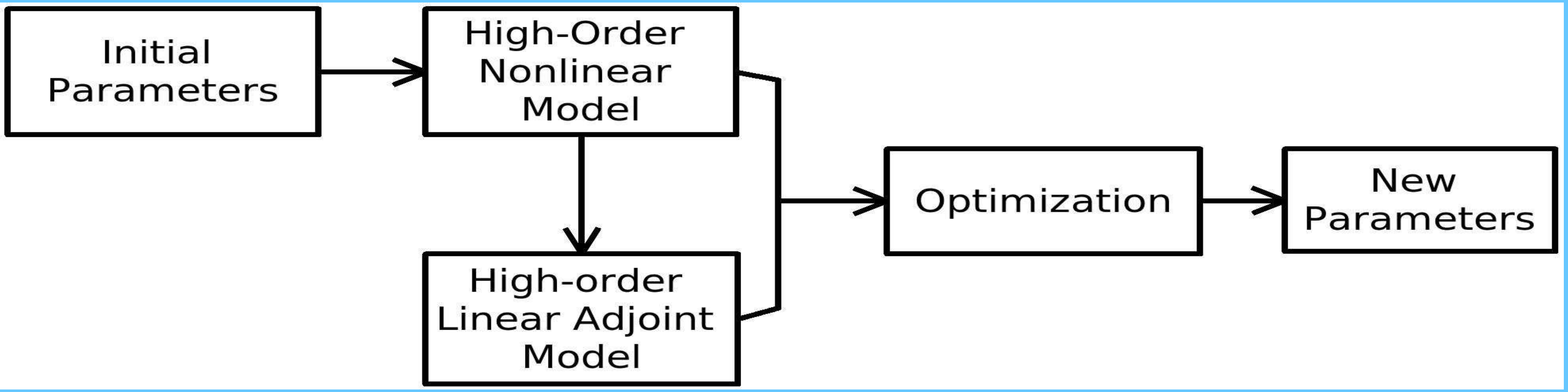
Outline

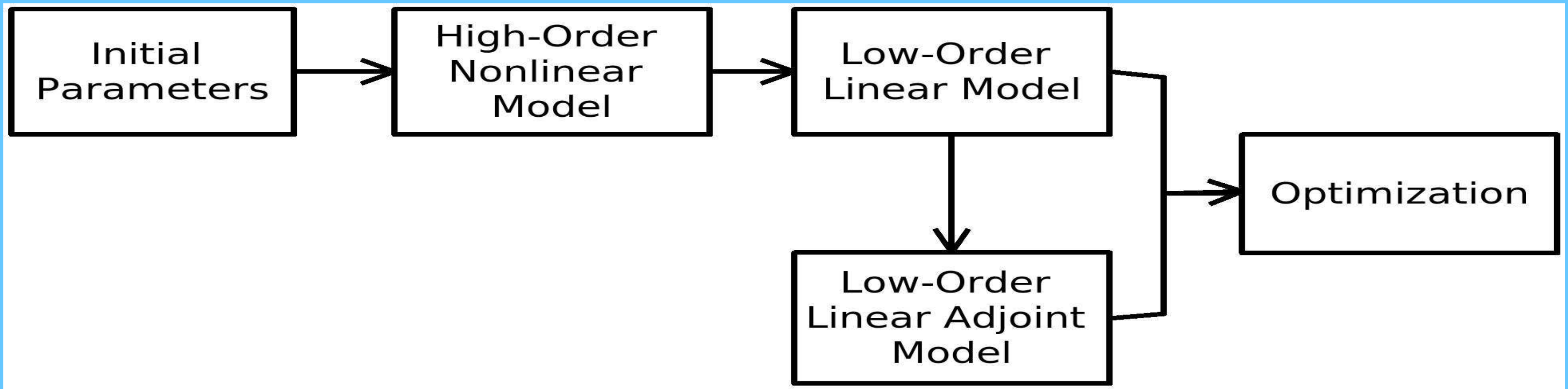
- Motivation
- POD Reduced Order Modeling
- Ensemble Approach
- The Dutch Continental Shelf Model DCSM
- Experiment and Results
- Conclusions

Motivation

- Adjoint Method:



- POD Method:



Vermeulen, P.T.M., Heemink, A.W. [2006]

Proper Orthogonal Decomposition (POD)

- Statistical tool to analyze experimental data:

  The POD is used to analyze the set of realizations with a view to extracting dominant features and trends (coherent structures called patterns in space)

- Reduced Order Modeling (ROM):

  The POD is used to provide a relevant set of basis functions with which we can identify a low-dimensional subspace on which to construct a model by projection of the governing equations

POD

- A set of s snapshots $E = \{e_1, e_2, \cdots, e_s\} \epsilon \Re^n$ are collected for some physical process taken at position e.

- Construct the covariance matrix Q $\epsilon \Re^{n \times n}$

$$Q = EE^T \tag{1}$$

- $P = \{p_1, p_2, p_3, \cdots\}$ are eigenvectors of a $n \times n$ eigenvalue problem with eigenvalues $\lambda_1 \gg \lambda_2 \gg \lambda_3 \cdots$

- Select the most dominant eigenmodes (patterns) based on the dominant eigenvalues λ_i

Ensemble Approach

- An ensemble of snapshot vectors of the forward model simulations is collected.
- The snapshots are perturbations with repect to estimated parameters $\boldsymbol{\gamma}_k$;

$$e_k(t_i) = \frac{\partial M_i[\mathbf{x}^b(t_{i-1}), \gamma_k]}{\partial \gamma_k} = \frac{M_i[\mathbf{x}^b(t_{i-1}), \gamma_k + \Delta\gamma_k] - M_i[\mathbf{x}^b(t_{i-1}), \gamma_k]}{\Delta\gamma_k} \tag{2}$$

- A reduced POD basis is obtained on the basis of this ensemble.

Ensemble Approach

- The reduced basis P is used to obtain approximate objective function:

$$J(\Delta\gamma) = \eta^T B^{-1}\eta + \sum_{i=1}[\{\mathbf{y}(t_i) - H(\mathbf{x}^b(t_i))\} -$$
$$\bar{H}\xi(t_i, \Delta\gamma)]^T R^{-1}[\{\mathbf{y}(t_i) - H(\mathbf{x}^b(t_i))\} - \bar{H}\xi(t_i, \Delta\gamma)] \quad (3)$$

ξ is a reduce time-varing state vector;

$$\begin{pmatrix} \xi(t_i) \\ \Delta\gamma \end{pmatrix} = \begin{pmatrix} \widetilde{M}_i & \widetilde{M}_\gamma \\ 0 & I \end{pmatrix} \begin{pmatrix} \xi(t_{i-1}) \\ \Delta\gamma \end{pmatrix} \quad (4)$$

$\widetilde{M}_i$ and $\widetilde{M}_\gamma$ are reduced dynamics operators which are computed as:

$$\widetilde{M}_i = P^T \frac{\partial M_i}{\partial x^b(t_{i-1})} P \quad (5)$$

$$\widetilde{M}_\gamma = P^T \left(\frac{\partial M_i}{\partial \gamma_1}, \cdots, \frac{\partial M_i}{\partial \gamma_u}\right) \quad (6)$$

Ensemble Approach

- We compute the jacobian $\frac{\partial M_i}{\partial x^b}$ by perturbing the nonlinear operator M_i along pattern direction.

$$\frac{\partial M_i}{\partial x^b(t_{i-1})} p_h = \frac{M_i[x^b(t_{i-1}) + \varepsilon p_h, \gamma_k] - M_i[x^b(t_{i-1}), \gamma_k]}{\varepsilon} \tag{7}$$

- Now the reduced dynamics operator $\widetilde{M}_i$ is obtained as:

$$\widetilde{M}_i = P^T \left(\frac{\partial M_i}{\partial x^b(t_{i-1})} p_1, \cdots, \frac{\partial M_i}{\partial x^b(t_{i-1})} p_r \right) \tag{8}$$

- The dimension of reduce model is smaller than that of original model.

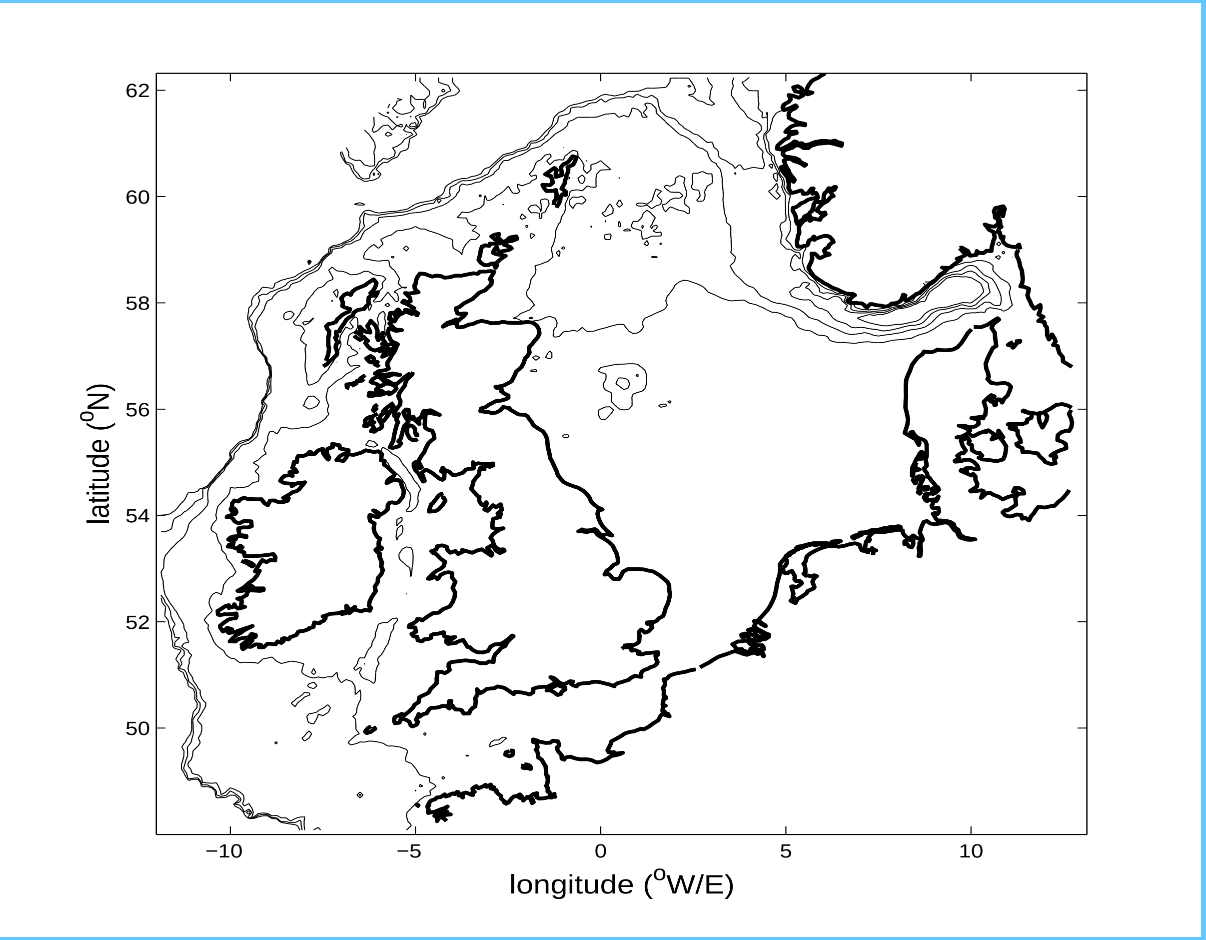
- Reduced model has linear characteristics. So it is easy to build a adjoint model for the computation of gradient.

The DCSM(v5)

- Large part of the area lies below mean sea water level
- 1 Feb 1953: severe storm surge, casualities in southwestern part
- Delta project: dikes, moveable surge barriers at the entrance of Harbor
- Water level prediction system

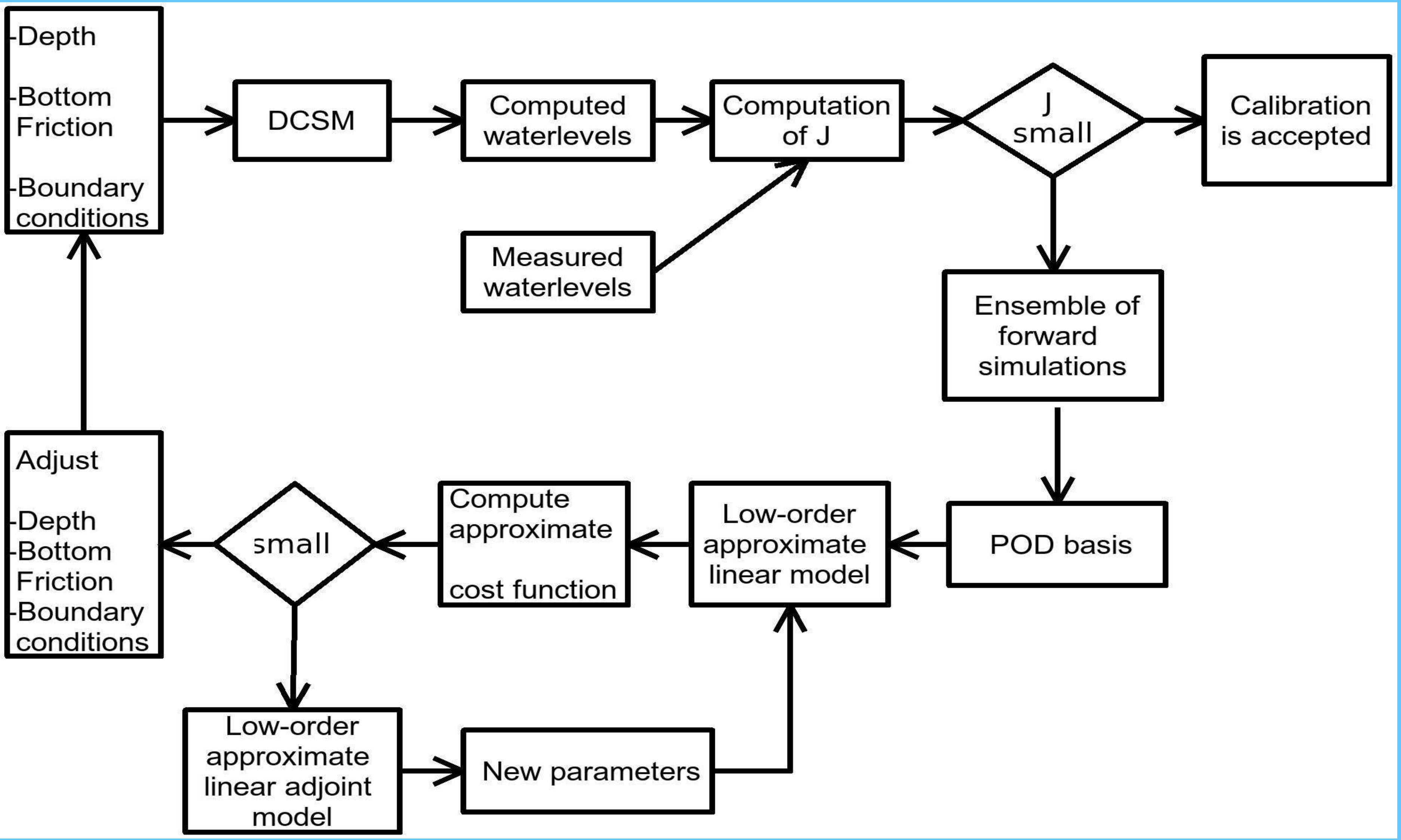
Model Area

- Around 20, 000 grid points
- Based on Shallow water equations



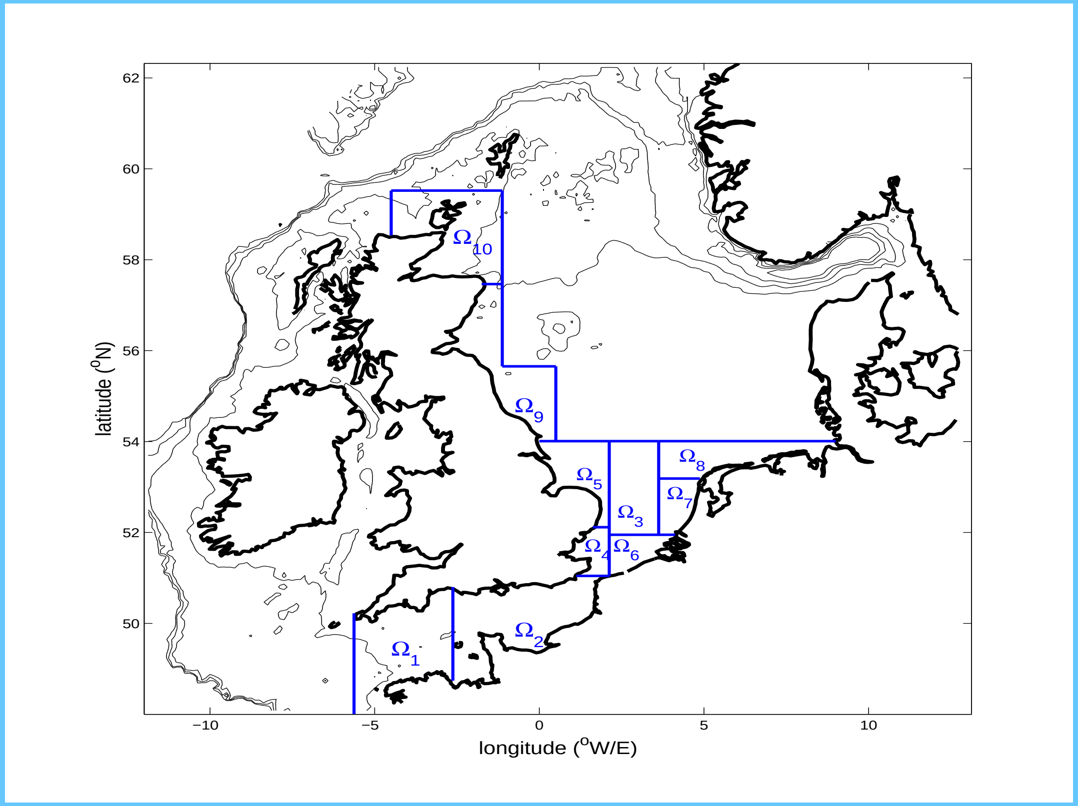
DCSM(v5)

- One outer iteration(β) with POD based calibration method:



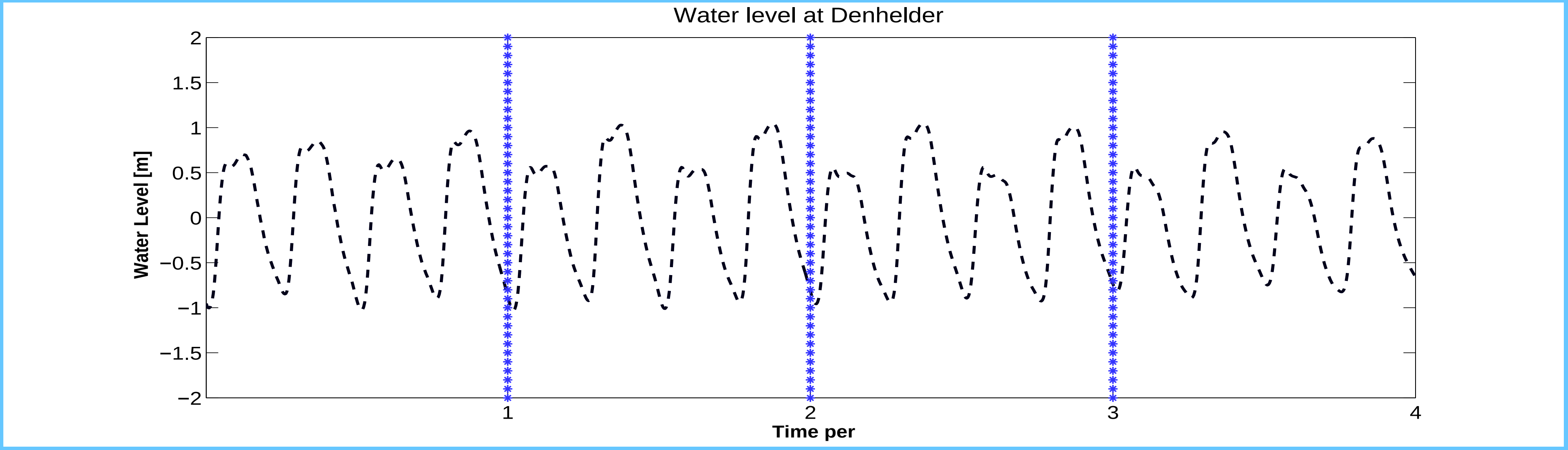
Experiment

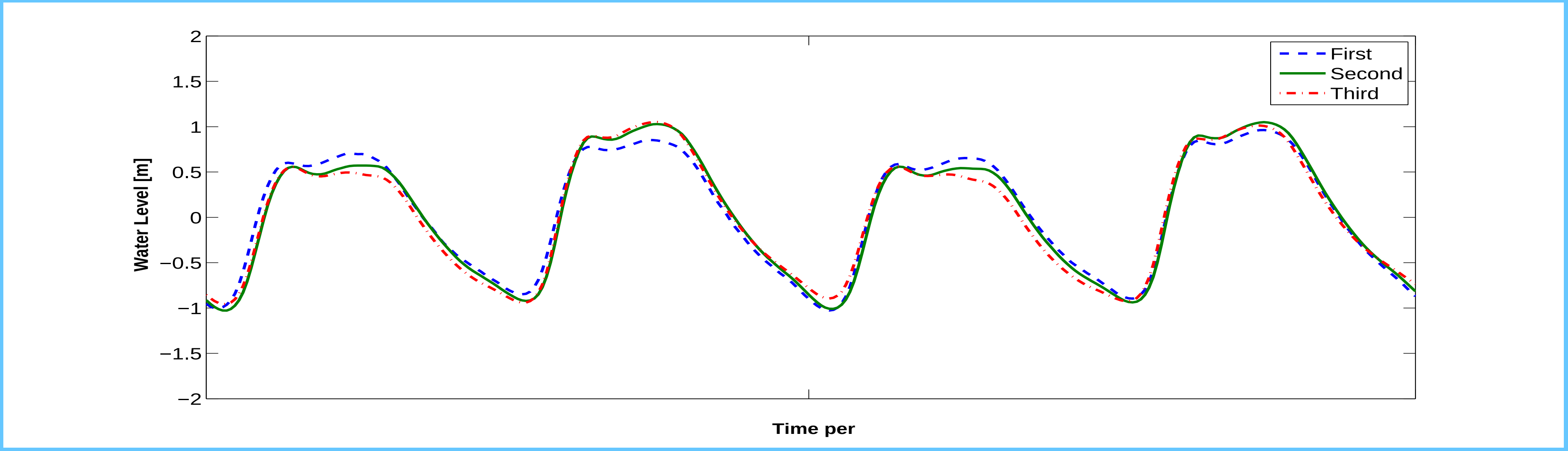
- Calibration run: 29 Dec 2006 to 30 Jan 2007
- Measurement data are used from : 01 Jan 2007 to 30 Jan 2007
- includes two spring-neap cycles.
- Assimilation stations: 24 Validation stations: 12
- No. of parameters: 13 Depth: 10 Bottom Friction: 3



Experiment

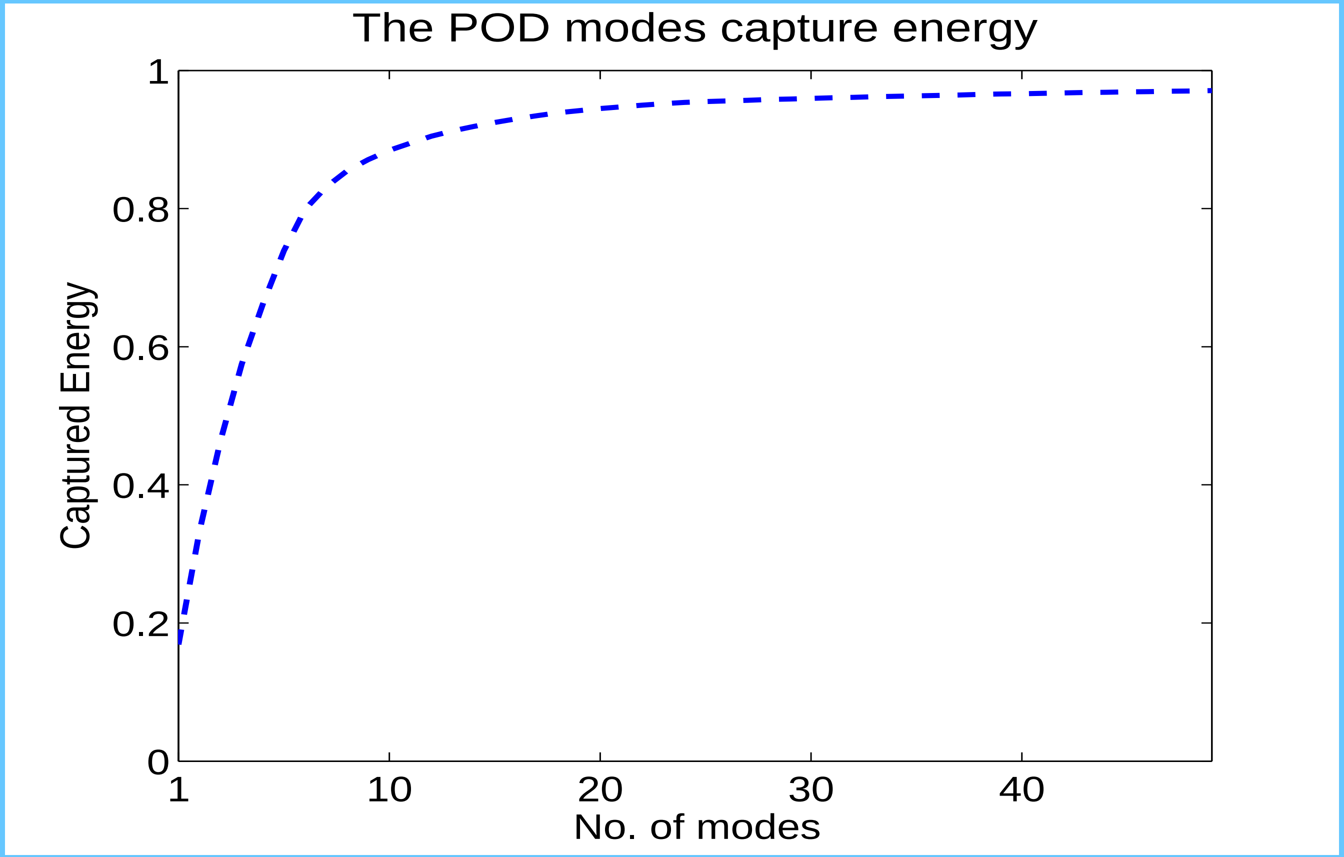
- Waterlevel timeseries at Den Helder





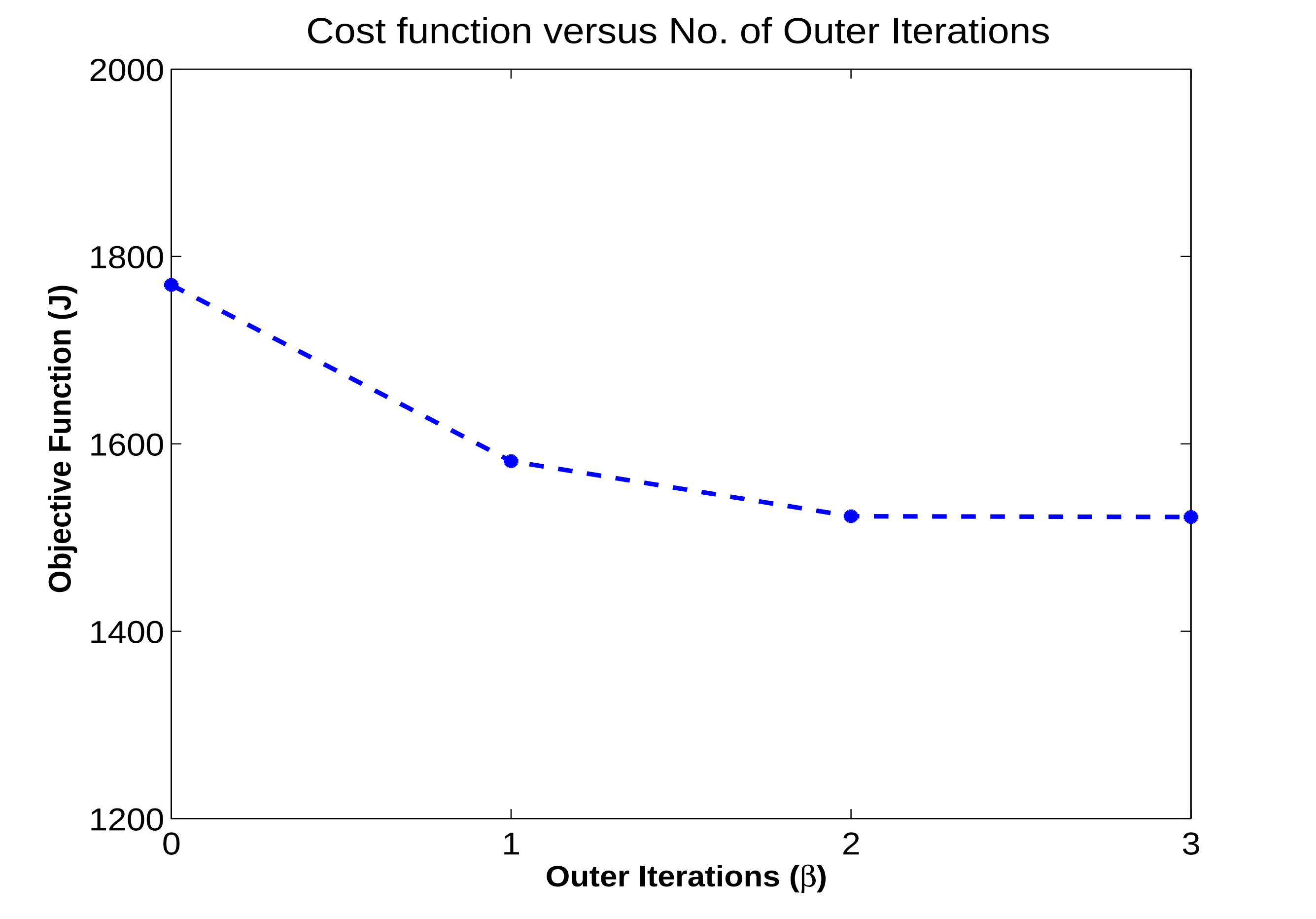
Experiment

- Ensemble: based on forward model simulations of 1st four days : 01 Jan 2007 to 04 Jan 2007
- Each snapshot vector contains the waterlevels h, velocities u and v.
- Ensemble size: 390 snapshot vectors



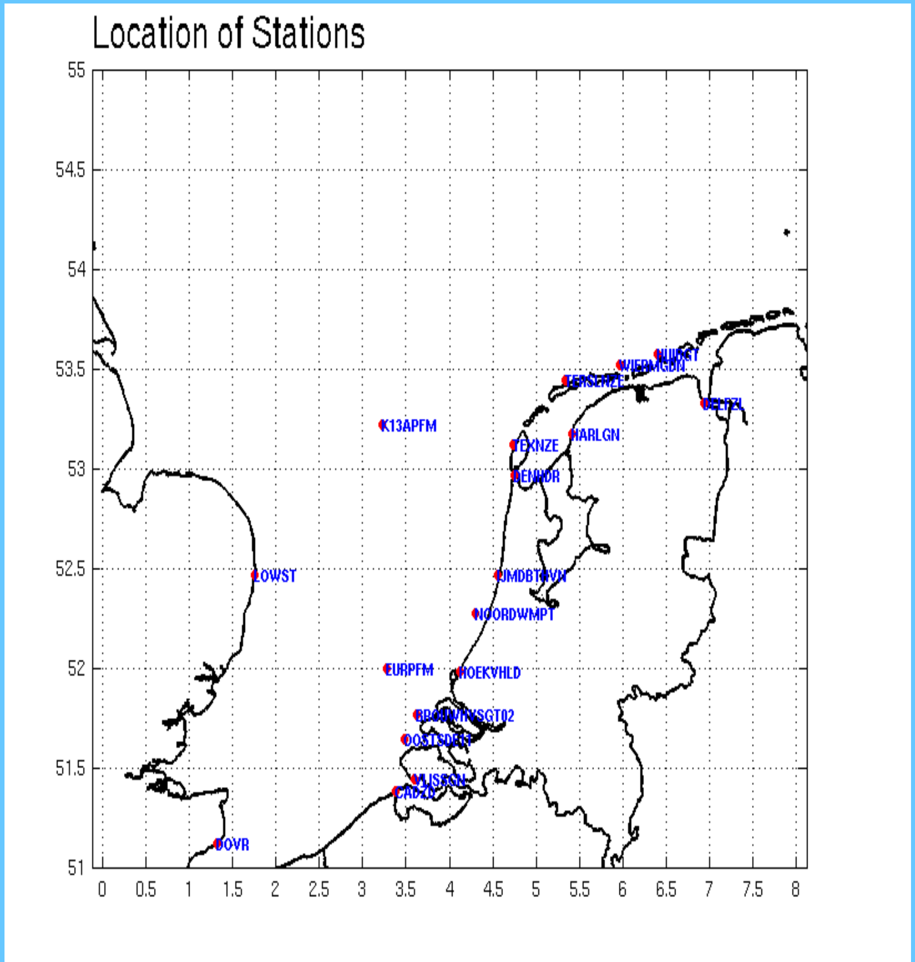
Results

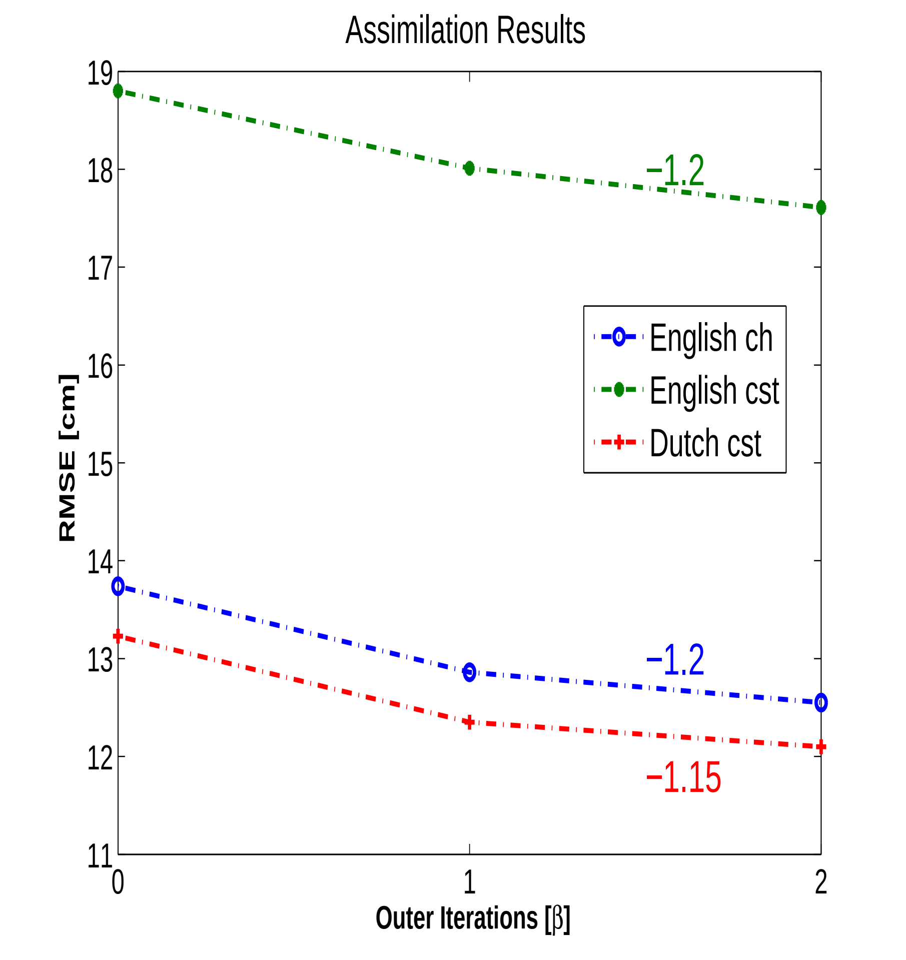
- A reduced model is formed with 95% captured energy (24 POD modes)
- The reduced model operates on $\Re^{24+13}$
- A 10% reduction in the cost function after one outer iteration.



Assimilation Results

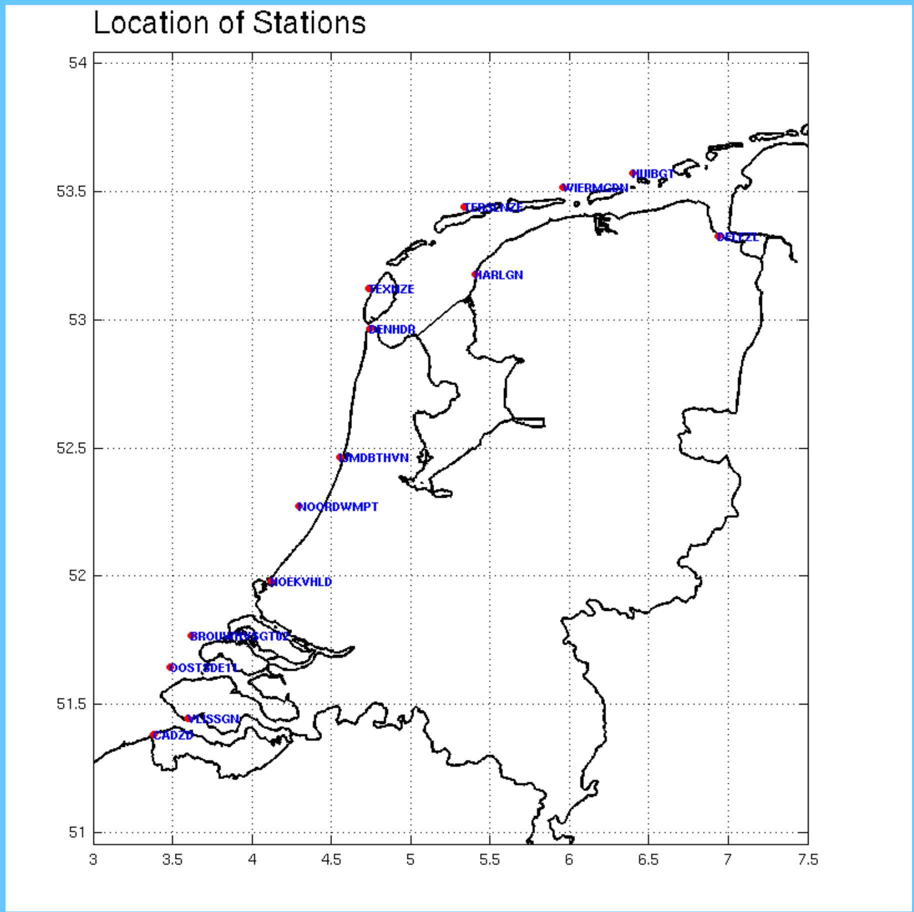
- English channel: 18 assimilation stations are used
- An overall improvement of $1.2cm$ is found in the English Channel after 2nd outer iteration

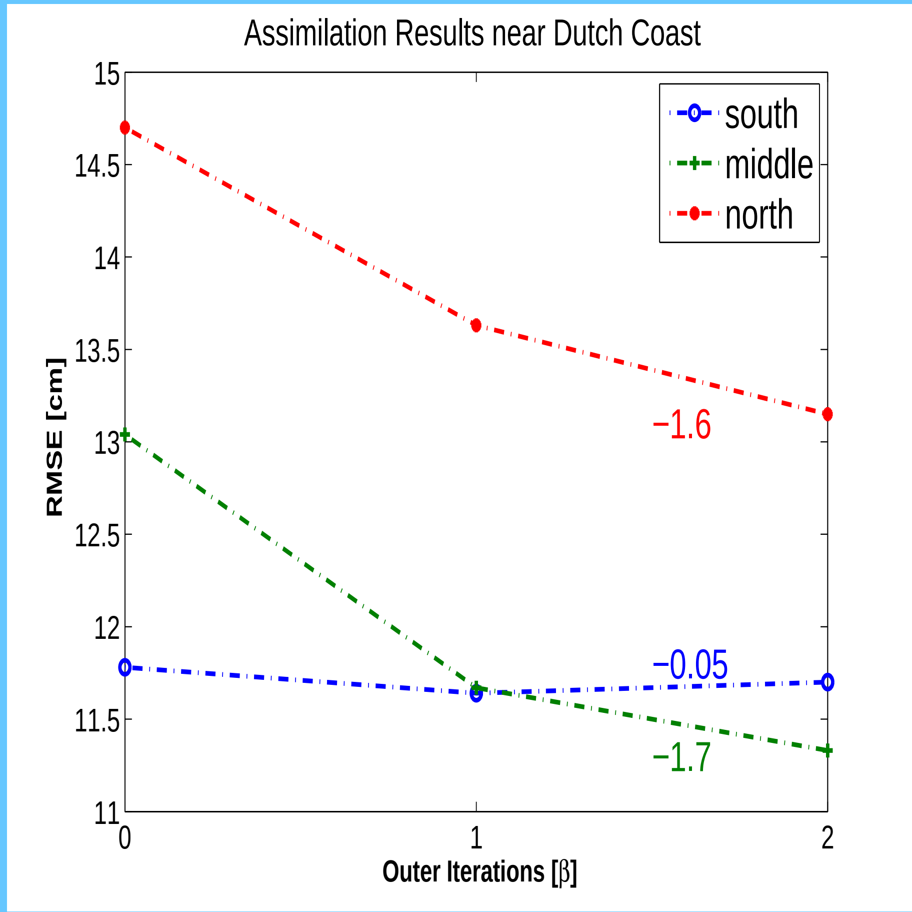




Assimilation Results

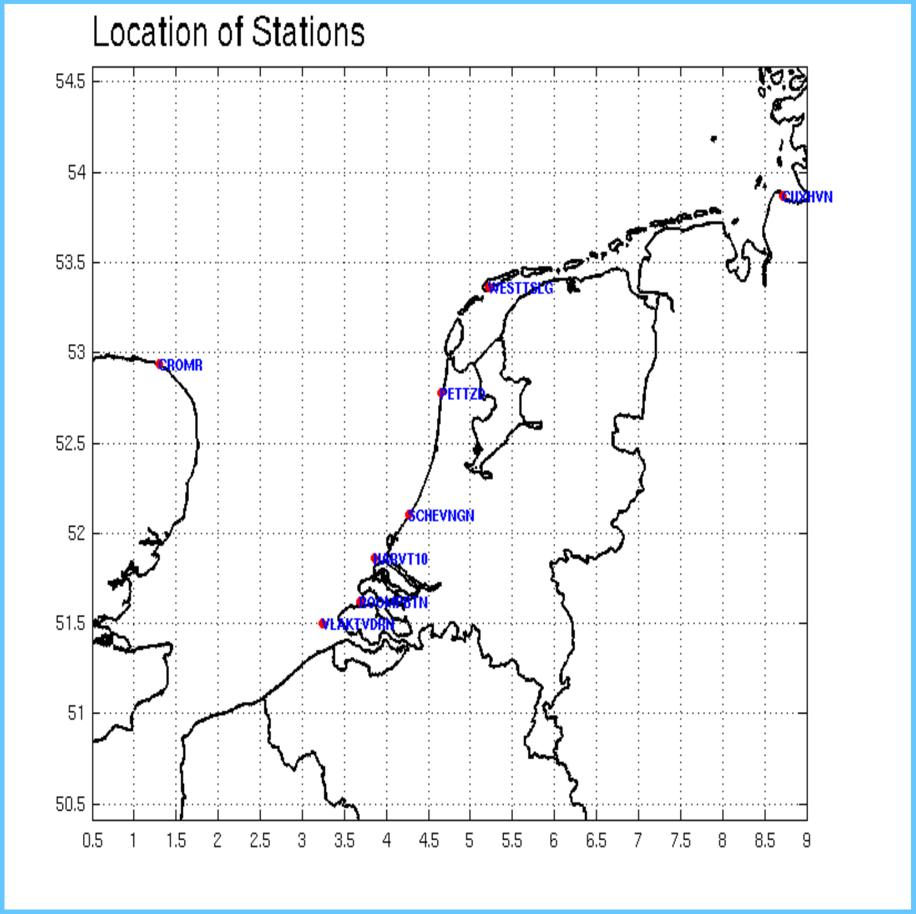
- A significant improvement is found in the north and middle regions of the Dutch coast
- A slight improvement in the southern region.

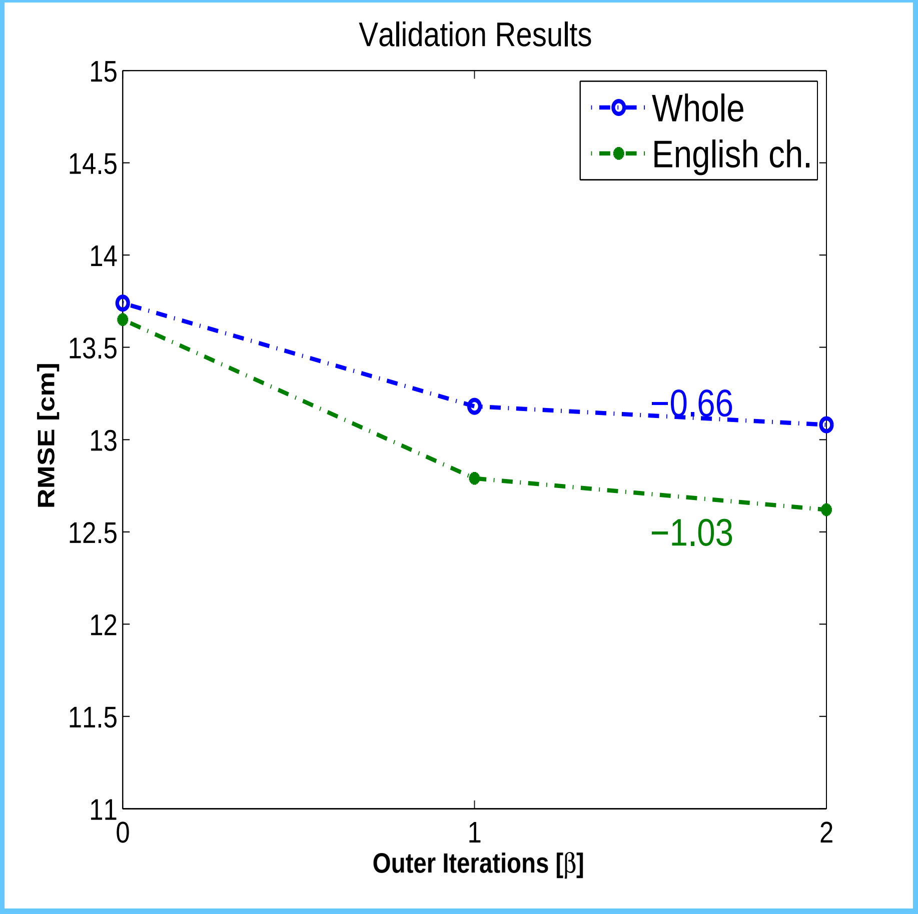




Validation Results

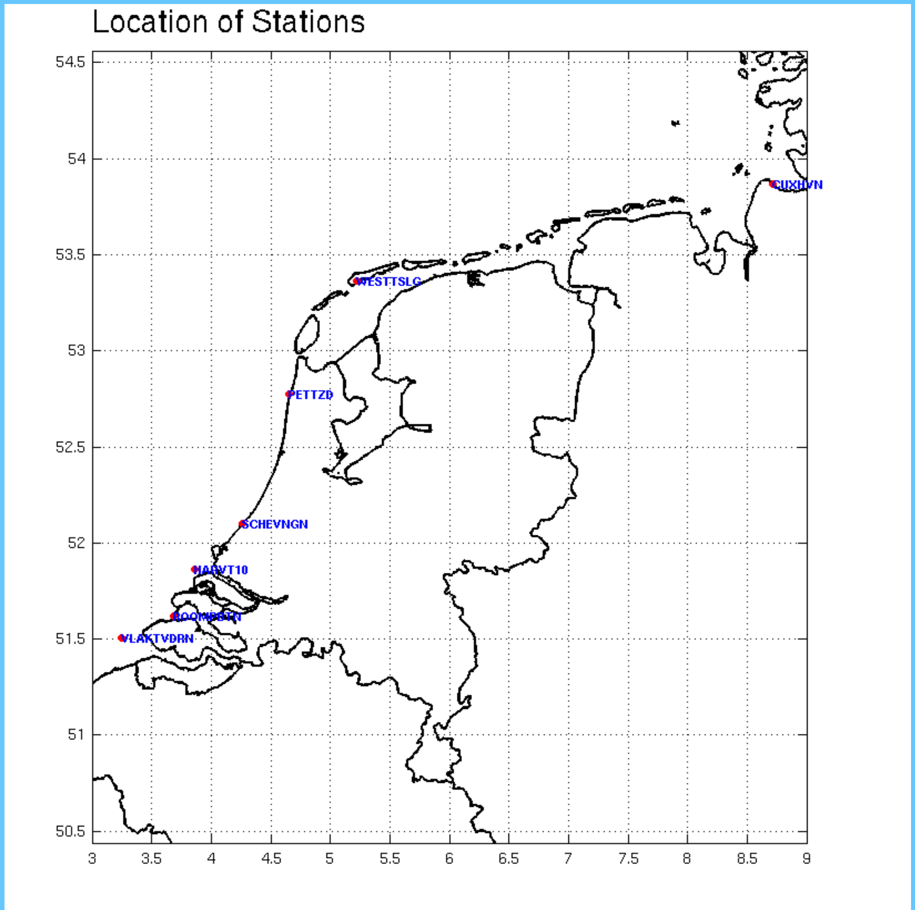
- English channel: 8 validation stations are used
- An overall improvement of $1.0cm$ is found in the English Channel

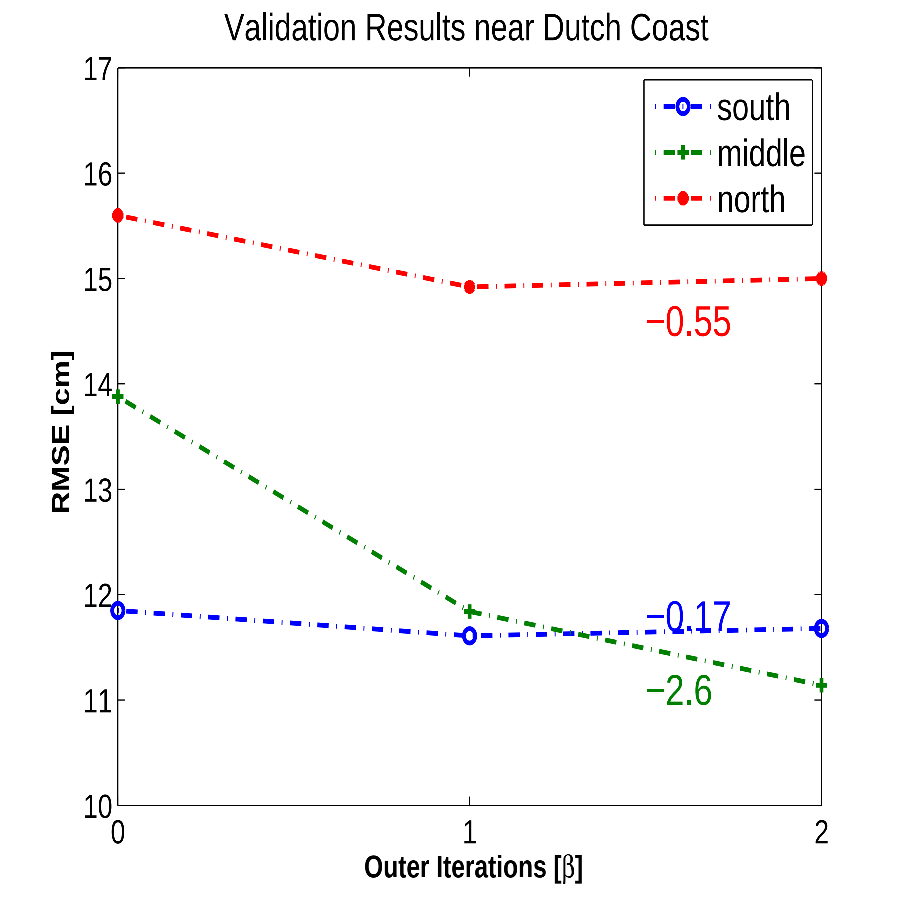




Validation Results

- A significant improvement is found in the middle region of Dutch coast
- No improvement in the northern region during 2nd outer iteration
- Again a slight improvement in the southern region.





results(contd)

- Computational cost of the algorithm:
- Number of parameters: 13

The computational cost is given in terms of No. of simulations of the original DCSM model.

background cost function : 3
Ensemble collection(only once): 3
Reduced model formulation: 1/2
Optimization :negligible(1/20)

- So the computational cost of the entire optimization is < 7 model simulations.

Conclusions and Future Work

- Negligible Optimization cost with the POD based model reduction technique.
- Classical method, adjoint of tangent linear model
- POD based method gives adjoint of linear reduce forward model
- Adjoint method gives exact gradient, more accurate
- POD based method gives approximate gradient.
- The POD method is dependent on the number of parameters. If the number of parameters are too large, the size of ensemble is too big and it is difficult to find a good approximate model.
- The cost of ensemble in each outer iteration can be reduced by using the same ensemble.

THANK YOU