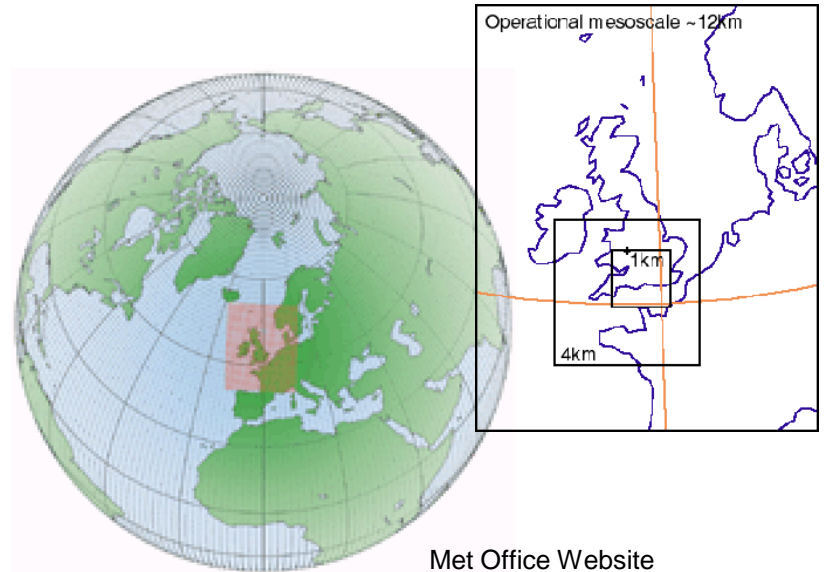


Issues in High Resolution Data Assimilation



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National Centre for Earth Observation

NATURAL ENVIRONMENT RESEARCH COUNCIL

NCEO: Delivering world-class science by unlocking the full potential of Earth Observation to monitor, diagnose and predict environmental and climate change, and ensuring that scientific advances translate into public good.

T1: EO for climate diagnosis and prediction (Integrating Theme)



- Exploiting EO to improve national capability for climate prediction over timescales from months to decades
- Reducing and quantifying uncertainty

Leaders: Prof K Helnes (Reading)
Prof A. Slingo (Reading)
Dr S Laxon (UCL)
Prof P Cox (Exeter)

T2: Monitoring, diagnosis, re-analysis and prediction of the global carbon cycle



- Understanding the feedbacks between physical and biological processes involving the carbon cycle, in order to predict changes in carbon fluxes at the Earth's surface

Leaders: Prof S Quegan (Sheffield)
Prof J Aiken (RIU)

T3: Atmospheric composition: air quality and climate



- Developing an integrated approach to the analysis of satellite measurements, to provide new information on atmospheric composition and aerosols for air-pollution forecasting and testing climate models

Leaders: Dr B Kerridge (RAL)
Prof M Chipperfield (Leeds)

T4: High resolution predictions of hazardous weather, floods and water resources



- Developing capability to forecast hazardous weather and hydrological consequences
- Understanding multiscale dynamics
- Developing novel assimilation techniques for highly non-linear processes

Leaders: Prof R Gurney (U of Reading)
Dr S Dance (U of Reading)

T5: Cryosphere and polar oceans



- Using new EO data to quantify changes in the mass balance of the cryosphere and to develop new models to represent the relevant processes in coupled climate prediction models
- Determining the impact of polar melt on the circulation of the ocean

Leader: Dr S Laxon (UCL)

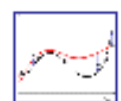
T6: Dynamic Earth and geo-hazards



- Using global satellite measurements of the Earth's surface and volcanic gas emissions to advance knowledge of processes responsible for earthquakes, tsunamis and volcanoes, and hence developing better warning systems

Leader: Prof B Parsons (Oxford)

T7: Data assimilation and treatment of uncertainty (Cross Cutting Theme)



- Developing the theory of data assimilation, including methods to treat data and model uncertainty, to underpin applications in NCEO and partner agencies
- Promoting collaboration with groups funded by the EPSRC on research into the underpinning theory

Leader: Prof I Roulstone (Surrey)

EO informatics (Underpinning Theme)



- Exploiting and developing e-science and new data informatics technologies in order to make EO data and derived products from multiple data centres more easily accessible, and to ensure proper archiving of data in line with NERC policy

Leaders: Dr B Lawrence (RAL)
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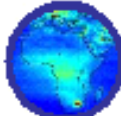
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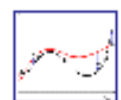
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Outline

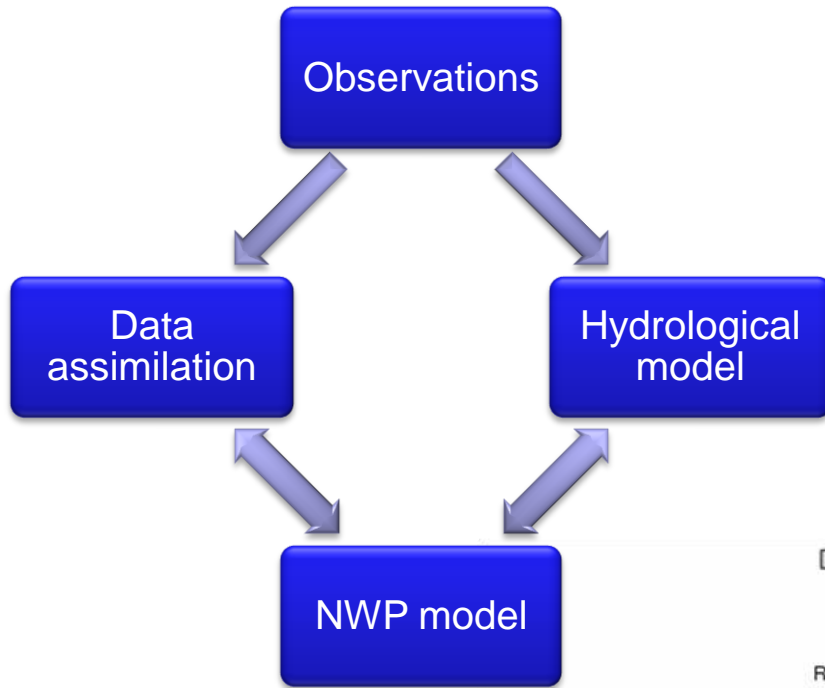
- Motivation
- Challenges
- Multi-scale Modelling
- Summary and Outlook

Hazardous weather and flooding



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Birmingham tornado 2005



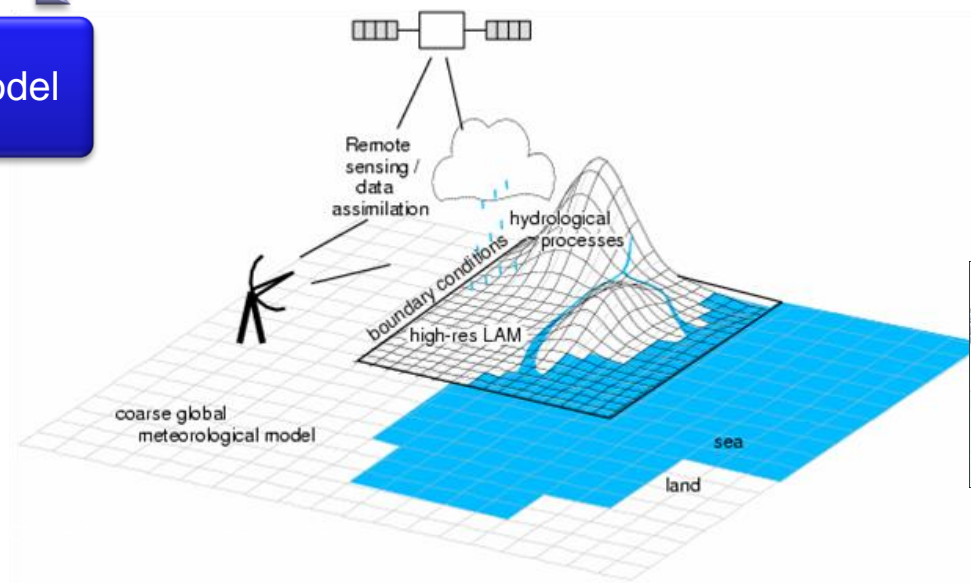
Heathrow fog, Christmas 2006



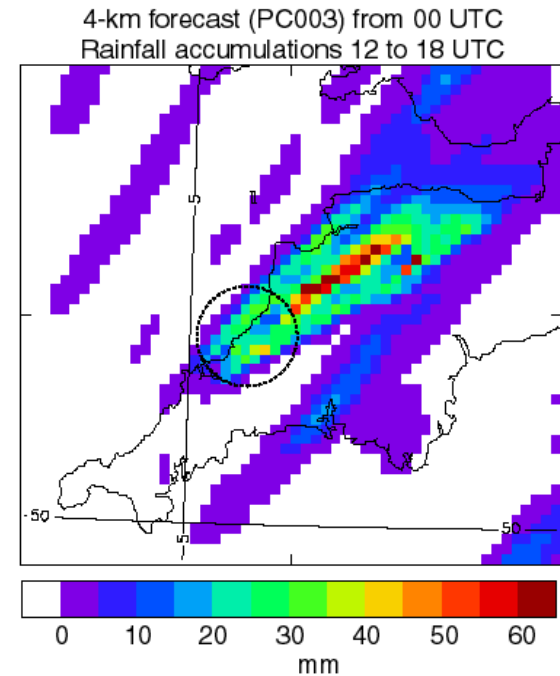
Boscastle storm 2004



Snow, 2009



- New observation types providing detail on required scales
- Operational storm-scale (1.5km) limited area models now expected – possibly higher resolution in future.



- Improvements in hydrological models, including increased interest in the use of more sophisticated data assimilation techniques.

Data assimilation on convective scales is a **NEW** problem – very different in character from assimilation on synoptic scales.

What are the challenges?

Challenges

1. Observations
2. Background Covariances
3. Multi-scale Dynamics / Coupled Systems
4. Nonlinearity and Uncertainty
5. Model Reduction

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Multi-Scale Dynamics

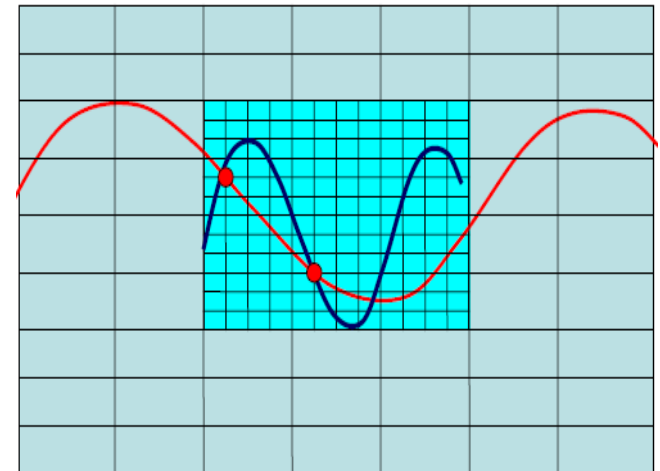
Strong dynamical forcings and **feedback** exist between **synoptic** and **storm-scale** systems. In **high resolution** convective models:

- **need** to update **fine-scale** information while preserving **large scale** information
- **need** lateral boundary conditions for nested **limited area** models from synoptic-scale data
- **need** to retain rapid **convergence** of all important scales in the optimization algorithm

Question:

How are different scales treated in a LAM?

- Study **aliasing** problems in limited area models: examine how different wave lengths are **projected** onto the **limited area analysis**, using a simple nested advection-diffusion model.
- Examine methods for combining **longer** wave-lengths from the **global** model with **shorter** wave-lengths from the **LAM**.

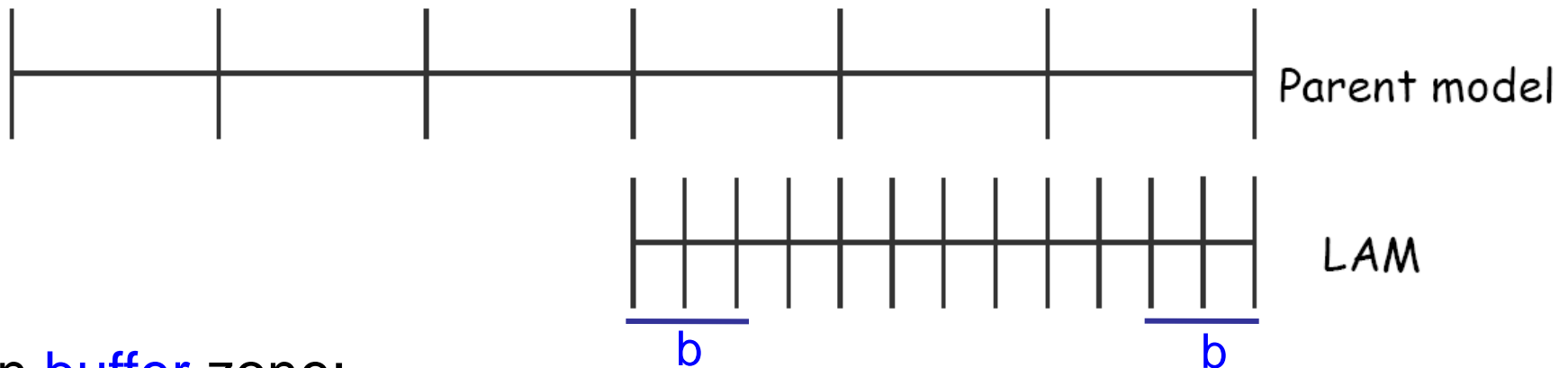


Model

The 1D linear advection-diffusion equation

$$U_t + CU_x = \sigma U_{xx}$$

with **periodic** boundary conditions for the **parent** model and the **parent analysis** for the **LAM** boundaries. Discretization is **explicit** time, **up-wind** advection and **centred** diffusion.



In **buffer** zone:

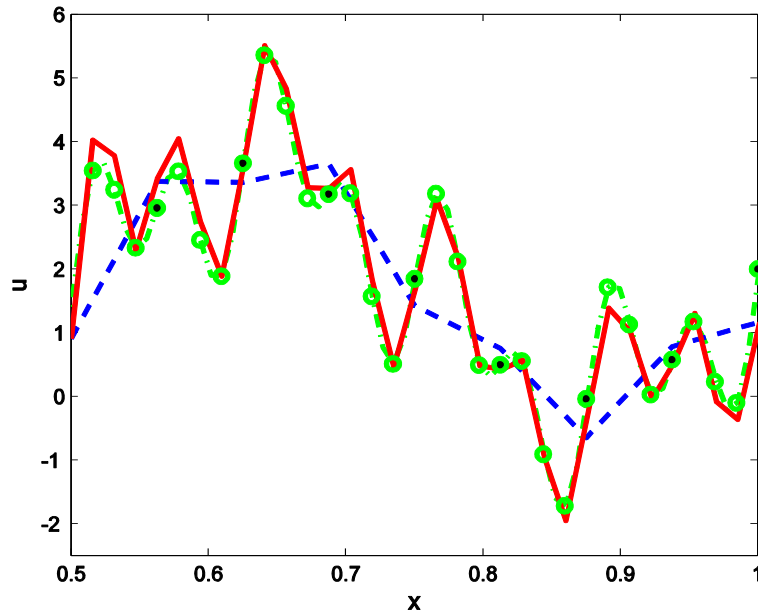
$$U_i^{new} = (1 - \omega) U_i^L + \omega U_i^G, \quad \omega = 1 - [(i - 1) / b]$$

Assimilation System

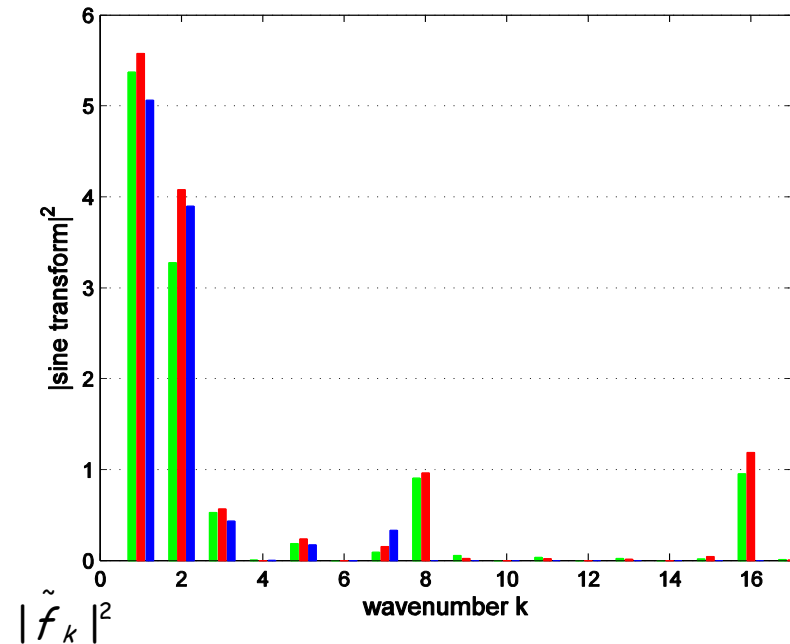
- Uses 4DVar
- Transforms to spectral space using double-sine control variable transform
- Perfect observations at all points
- LAM boundary conditions from parent analysis
- Davies Relaxation at LAM boundaries
- High Resolution LAM = 4 x parent
- High Resolution truth = 2 x LAM

Experiment 1 : Long and short waves

$$\text{truth} : 2\sin(x/4) + 2\sin(2x) + \sin(8x) + \sin(16x)$$



LAM domain



Power spectrum

o observations --- truth --- parent analysis --- LAM analysis

Summary:

- Higher resolution allows **higher wave-numbers** to be **captured** by the LAM
- A large proportion of the “**long wave**” information is **aliased** onto wave-number **$k=1$**
- Some “long wave” information is aliased onto higher wave-numbers
- These conclusions can be shown to hold **mathematically** for a general case using discrete Fourier transforms

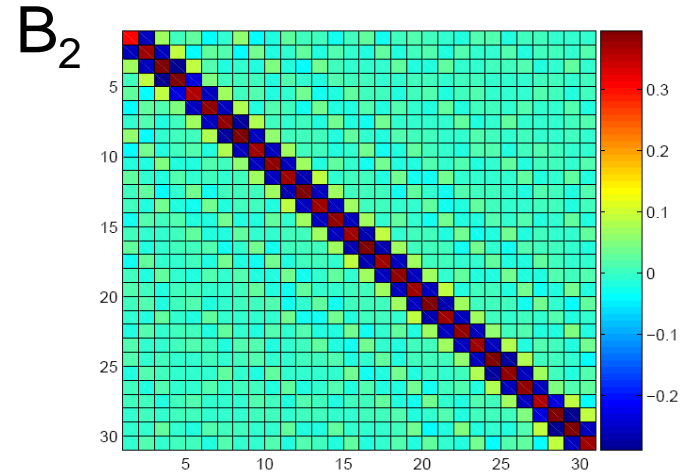
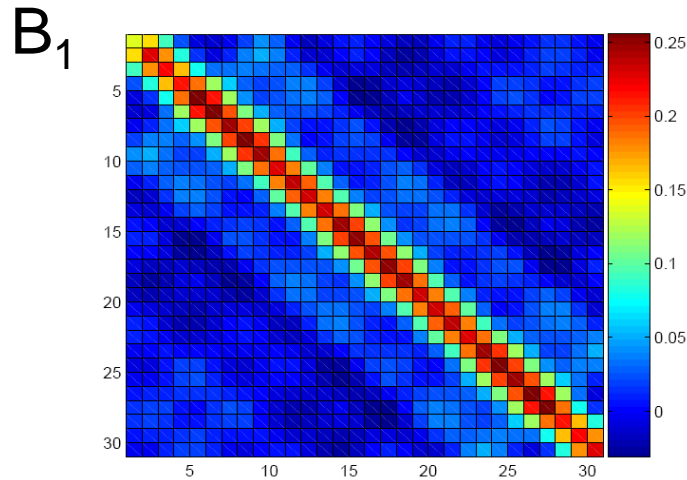
Assimilation in Spectral Space

$$\mathcal{J}(\mathbf{x}_0) = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^T \mathbf{B}^{-1}(\mathbf{x}_0 - \mathbf{x}_b) + \frac{1}{2} \sum_{k=0}^T (\mathbf{y}_k - h_k(\mathbf{x}_k))^T \mathbf{R}_k^{-1}(\mathbf{y}_k - h_k(\mathbf{x}_k))$$

sine transform: $\mathbf{x} = \mathbf{U}\mathbf{z}$, $\Sigma^{-1} = \mathbf{U}^T \mathbf{B}^{-1} \mathbf{U}$

$$\mathcal{J}(\mathbf{x}_0) = \frac{1}{2}(\mathbf{z}_0 - \mathbf{z}_b)^T \Sigma^{-1}(\mathbf{z}_0 - \mathbf{z}_b) + \frac{1}{2} \sum_{k=0}^T (\mathbf{y}_k - h_k(\mathbf{U}\mathbf{z}_k))^T \mathbf{R}_k^{-1}(\mathbf{y}_k - h_k(\mathbf{U}\mathbf{z}_k))$$

Background Matrix B



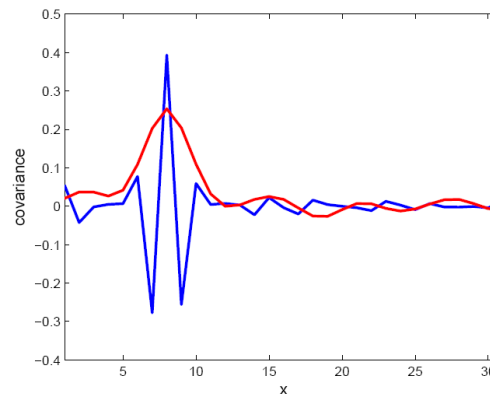
$$\Sigma_1 = \text{diag}\{1.0, 0.5, 0.1, 0.01, 0.005\}$$

$$\Sigma_2 = \text{diag}\{0.005, 0.01, 0.1, 0.5, 1.0\}$$

Correlation structure

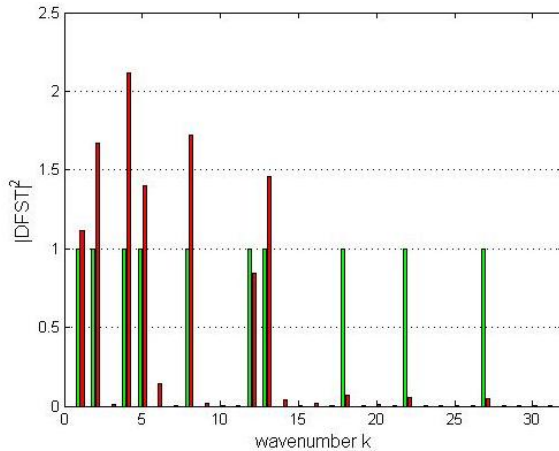
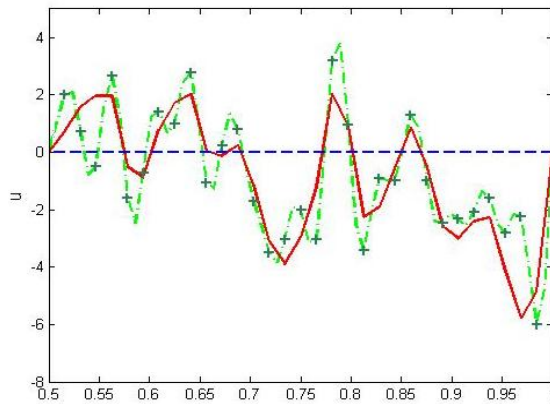
Red = B_1

Blue = B_2



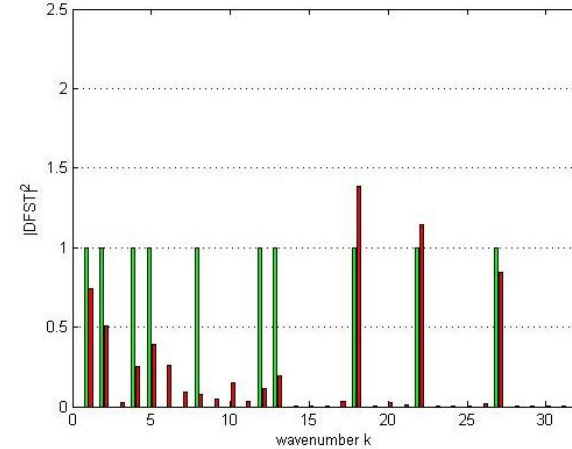
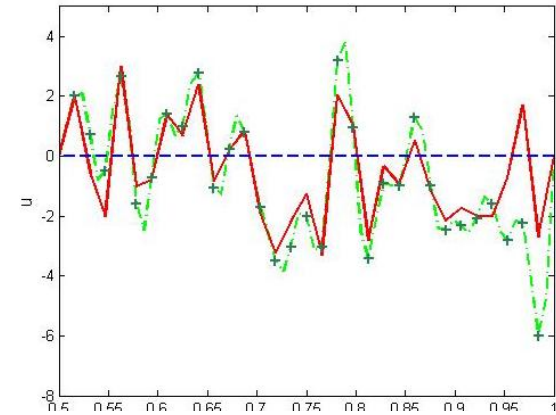
Different weightings in spectral space on background

$$\Sigma^{-1} : \left(\begin{array}{c|c} \mathbf{0} & \\ \hline & \mathbf{I} \end{array} \right)$$



LAM
domain

$$\Sigma^{-1} : \left(\begin{array}{c|c} \mathbf{I} & \\ \hline & \mathbf{0} \end{array} \right)$$



Power
spectrum

o observations --- truth --- parent analysis --- LAM analysis

Conclusions

- Wave-lengths **shorter** than the resolution of the global models can be **analysed** in the **LAM**, but **longer** wave-lengths may be **incorrectly represented** due to aliasing.
- Weighting **global** background differentially **in spectral space** can affect scales analysed in LAM model.

G.M. Baxter, *PhD Thesis*, 2009

Further Work - ???

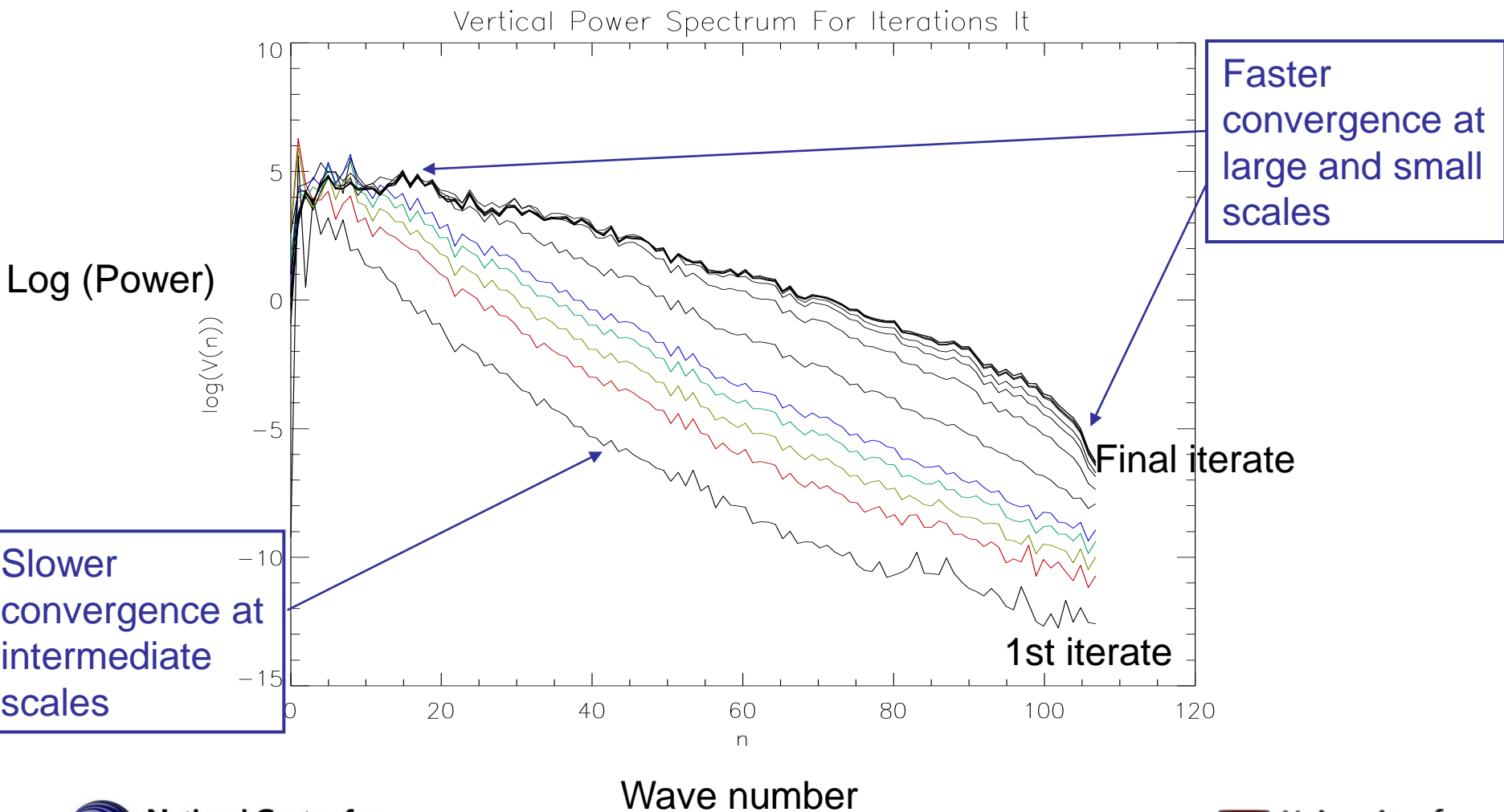
Further Work - ???

- Test methods for combining long wave information from Global models with high frequency information from the Lam via control variable transforms more generally
- Improved treatment of boundary conditions
- Scale-dependence of 4DVar convergence

Multi-scale systems (2)

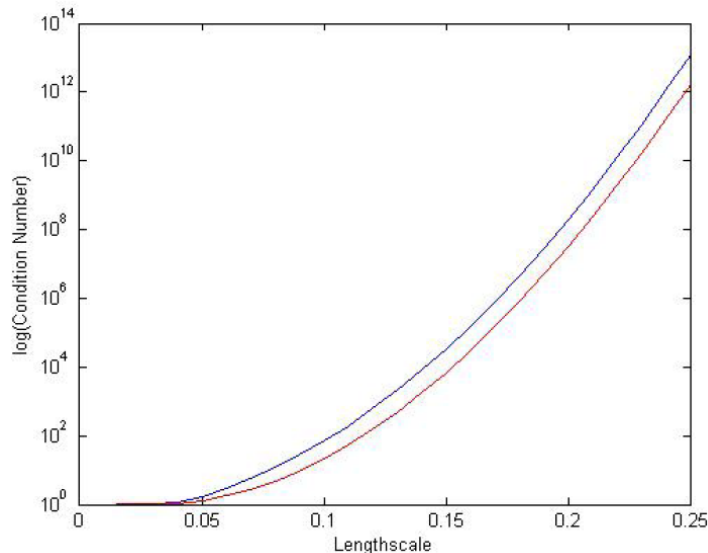
- **Convergence** of the inner loop of the **Met Office** incremental 4DVar data assimilation system **at different Fourier scales** has been analysed. Multi-level optimization methods are planned for development.
- **Conditioning** of the linearized minimization problem as a **function** of the **length-scales** in the **background covariances** and as a function of the observation variances.

Fourier spectrum of pressure increment at lowest model level as inner loop converges.



Conditioning of 3DVar

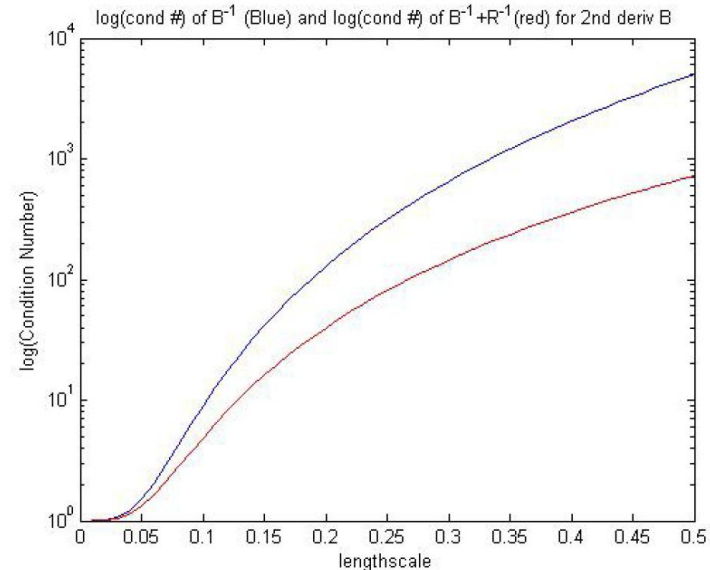
Condition Number of $(B^{-1} + HR^{-1}H^T)$ vs Length Scale



Periodic Gaussian Exponential

$$\mathbf{B}_{ij} = \sigma_b^2 \exp\left(\frac{-r_{i,j}^2}{2L^2}\right)$$

Blue = no obs Red = with obs variances 0.1 / 0.2



Laplacian 2nd Derivative

$$\mathbf{B}^{-1} = \gamma^{-1} \left(\mathbf{I} + \frac{l^4}{2\Delta x^4} (\mathbf{L})^2 \right)$$

Results:

- The **Met Office** inner loop **converges** more **slowly** at **mid-wave-lengths**. Multigrid approach might improve rates.
- **Conditioning** of inner linear system **decreases** with the **length scales** in the background error covariance matrix.
- **Conditioning** is **improved** by the addition of the **observations**

Haben et al., *Internal Reports*, 2009

The Discrete Fourier sine Transform of a function f_j is

$$\hat{f}_k = \sum_{j=1}^{N-1} f_j \sin(\pi j k / N)$$

where k is the wavenumber and N is the number of gridpoints

