The Sensitivity of Adjoint Sensitivity

Brian Ancell, Department of Atmospheric Sciences, U. of Washington

The Problem

Adjoint sensitivity is useful for a variety of applications, but exhibits significant variability with respect to various parameters such as:

- Model resolution
- Basic-state trajectory
- Model physics

The **goal of this study is to understand the "sensitivity of sensitivity"** toward the best use of sensitivity in data assimilation, observation targeting, and dynamical interpretation applications.

Sensitivity Variability Background

Adjoint sensitivity formulation for a forecast response function J:

$$\partial J/\partial Y_o = (\partial f(Y)/\partial Y_{o,t})^T * \partial J/\partial Y_t \qquad (1)$$

Adjoint Sensitivity variables — Tangent-linear propagator — Differentiated J w.r.t. forecast

- **dY/dt = f(Y)** is the nonlinear forward forecast model
- **Y** is a vector of model state variables

The above formula represents the adjoint sensitivity field about a single, previously-run forecast begun from initial condition Y_o.

The adjoint sensitivity about a 2nd forecast trajectory ($\partial J/\partial Y_o'$) begun from initial condition $Y_o + \Delta Y_o$ can be approximated by:

$$\partial J/\partial Y_o' = (\partial/\partial Y_{o,t}\,[f(Y_o)+\partial f/\partial Y_{o,t} * \Delta Y_o])^T * \partial J/\partial Y_t \qquad (2)$$

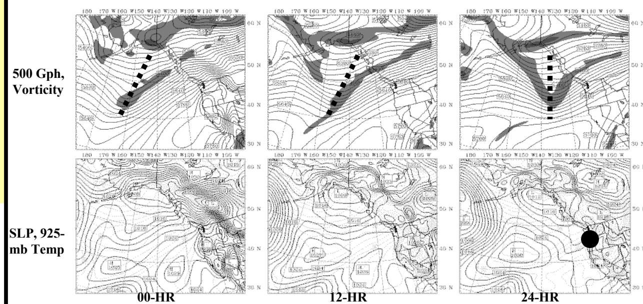
Subtracting (2) from (1) yields the difference in adjoint sensitivity calculated about 2 different forecast trajectories:

$$\Delta\partial J/\partial Y_o = (\partial/\partial Y_{o,t}\,[\partial f/\partial Y_{o,t} * \Delta Y_o])^T * \partial J/\partial Y_t \qquad (3)$$

- Linear **f(Y)** gives same sensitivity for all forecasts (**$\Delta\partial J/\partial Y_o=0$**)
- $\Delta\partial J/\partial Y_o$ for Nonlinear **f(Y)** depends on initial condition differences (**ΔY_o**) and the forecasts themselves (for nonlinearity greater than quadratic)

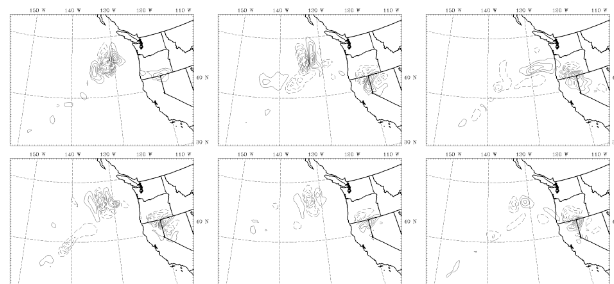
So how do sensitivities vary in practical forecasting applications?

The following plots illustrate a typical wintertime synoptic pattern impacting the Pacific Northwest U.S.:



In this scenario, a trough aloft moves toward the North American coastline, forcing a weak cyclone into the PNW at 24-hr forecast time.

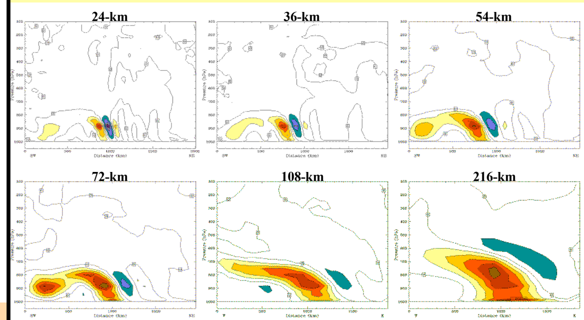
Using a **single-point SLP response function at 24-hr** at the black dot above, the adjoint sensitivity was calculated about all 90 equally-likely forecasts of the University of Washington Ensemble Kalman Filter (EnKF) for this case. The following plots represent the **sensitivity to 00-hr 850-mb temperature** calculated about 6 of these forecasts:



Solid=positive, dashed=negative, contour interval=0.05 Pa/K

- **Significant differences exist in structure, location, and magnitude** among the 6 equally-likely initial-condition sensitivity fields
- These differences result in a **large range of predicted response function perturbations** for identical initial-condition perturbations
- **Nonlinear perturbation evolution** about the EnKF mean in this case is to blame for these discrepancies, and reveals the **difficulty in diagnosing mesoscale regions where errors can grow rapidly**

Significant differences in sensitivity also arise when calculating **sensitivity at different horizontal resolutions**. For a similar synoptic situation and 24-hr SLP response function, the following plots depict vertical cross-sections of sensitivity to 00-hr temperature:



Warm colors=negative, cool colors=positive, contour interval varies

- **Differences are apparent in the horizontal scale, degree of vertical tilt, level of maximum sensitivity, and vertical extent** of the adjoint sensitivity fields
- The sensitivity field **magnitudes increase at higher resolution** and are about **50 times larger at 24-km resolution than at 216-km resolution**

Differences also exist using simplified adjoint physics:

Solid=positive, dashed=negative, contour interval=0.15 Pa/K

- **Simplified adjoint physics (dry in this case) appear to be the limiting factor,** causing sensitivity to resemble that of the totally dry case, even with a moist forward trajectory

Conclusions

- Much like an ensemble of forecasts, **adjoint sensitivity should be thought of probabilistically**
- **Significant differences in the sensitivity field can occur using different model parameters like resolution or physics,** and these issues should be considered as possible sources of error in various adjoint applications