Separating the wheat from the chaff – disambiguating dynamical interpretation of adjoint sensitivity fields

Michael C. Morgan and Brett T. Hoover
University of Wisconsin – Madison

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Focus

- Successful application of adjoint-derived forecast sensitivities to synoptic case studies requires a well-defined response function, *R*.

- Many synoptic case studies seek to understand how a particular feature or "disturbance" is influenced by its "environment", but separating the flow into disturbance and environment is non-trivial.

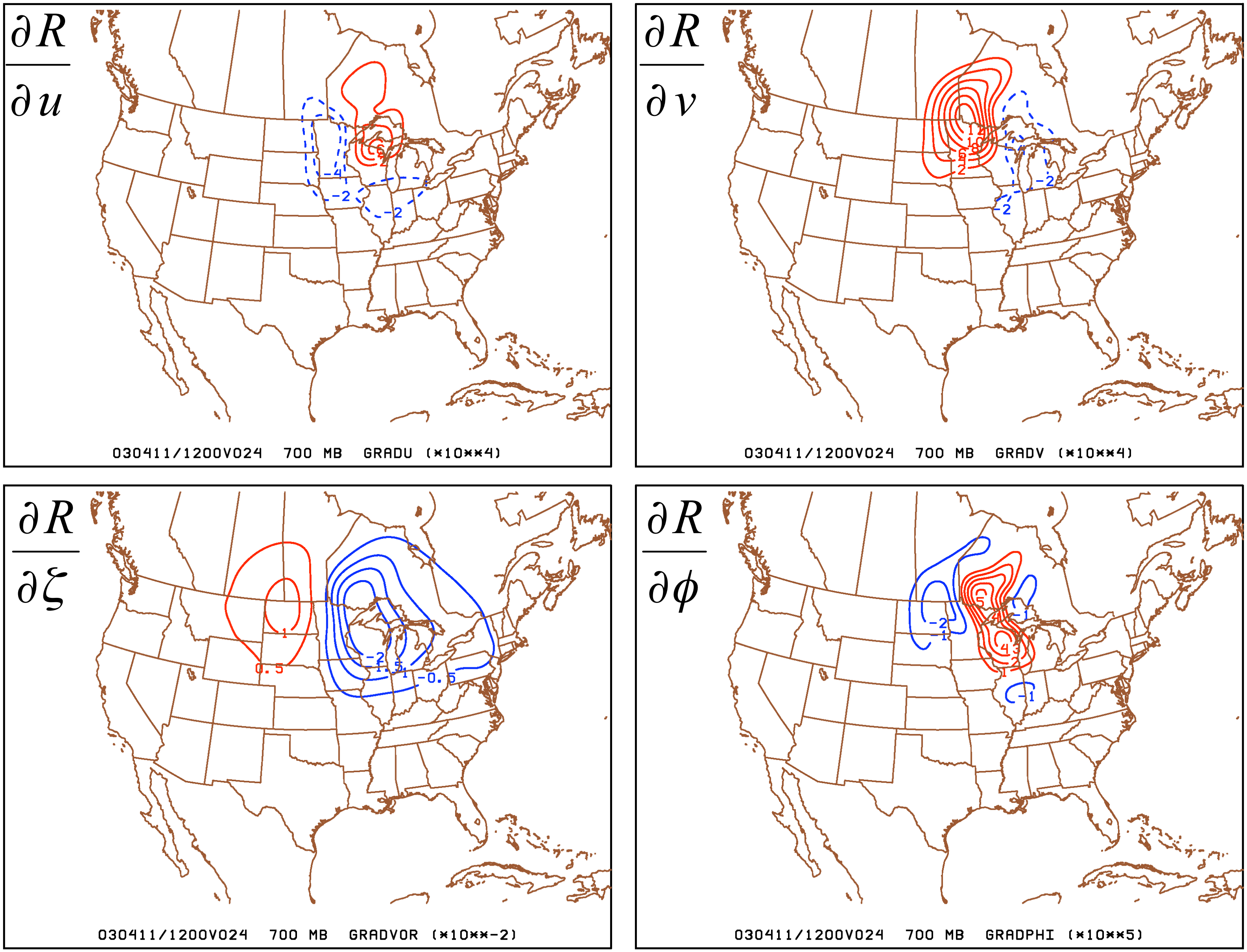
- Interpretation of these sensitivities is enhanced if the sensitivities are evaluated with respect to diagnostic, dynamical variables.

- The focus of this talk is on the use of potential vorticity (PV) to define a response function (for TC steering) and the evolution of sensitivities to PV in a shallow water model.

Why potential vorticity?

- **Conservation**
- **Invertibility**: One can solve for the wind and temperature perturbations associated with a given PV distribution (subject to prescribed balance constraints and boundary conditions)
- "Action at a distance": Flow at any point is attributable to the (potential) vorticity distribution at all other surrounding points
- When used to define a response function, PV allows for the unambiguous separation of "environment" from "disturbance"
- For response functions associated with balanced phenomena, results of interpretations of sensitivities calculated with respect to PV can be succinct.

700 hPa sensitivity gradients valid at 1200 UTC 11 April 2003 (f24)



Obtaining $\frac{\partial R}{\partial \zeta}$ and $\frac{\partial R}{\partial \delta}$ from $\frac{\partial R}{\partial u}$ and $\frac{\partial R}{\partial v}$

$$\nabla^2\left(\frac{\partial R}{\partial \zeta}\right) = -\left[\frac{\partial}{\partial x}\left(\frac{\partial R}{\partial v}\right) - \frac{\partial}{\partial y}\left(\frac{\partial R}{\partial u}\right)\right]$$

$$\nabla^2\left(\frac{\partial R}{\partial \delta}\right) = -\left[\frac{\partial}{\partial x}\left(\frac{\partial R}{\partial u}\right) + \frac{\partial}{\partial y}\left(\frac{\partial R}{\partial v}\right)\right]$$

Note that because of the elliptic property of the Laplacian operator, the sensitivities with respect to the vorticity and divergence will necessarily be of ***larger scale*** than the sensitivities with respect to the zonal and meridional components of the wind.

Linearized shallow water system

$$\frac{\partial u'}{\partial t} - fv' = -g\frac{\partial \eta}{\partial x}$$

$$\frac{\partial v'}{\partial t} + fu' = -g\frac{\partial \eta}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + H(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y}) = 0$$

For this system, the (locally) conserved dynamical variable is the linearized potential vorticity:

$$q' = \frac{\zeta'}{H} - \frac{f\eta}{H^2}$$

This system is solved numerically on an Arakawa C-grid using a leap-frog time stepping scheme. The model control variables are *u*, *v*, and η. The corresponding adjoint model is also written (ADJ of FD).

Diagnostic variables: *q*, *δ*, and *a*

$$\begin{pmatrix} q' \\ \delta \\ a \end{pmatrix} = \begin{pmatrix} -\frac{1}{H}\frac{\partial}{\partial y} & \frac{1}{H}\frac{\partial}{\partial x} & \frac{-f}{H^2} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ -\frac{\partial}{\partial y} & \frac{\partial}{\partial x} & -\frac{g}{f}\nabla^2 \end{pmatrix} \begin{pmatrix} u' \\ v' \\ \eta \end{pmatrix}$$

a is the ageostrophic vorticity:

$$a = \zeta' - \frac{g}{f}\nabla^2\eta$$

Diagnostic adjoint variables for *q*, *δ*, and *a*

$$\begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{\eta} \end{pmatrix} = \begin{pmatrix} \frac{1}{H}\frac{\partial}{\partial y} & -\frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ -\frac{1}{H}\frac{\partial}{\partial x} & -\frac{\partial}{\partial y} & -\frac{\partial}{\partial x} \\ -\frac{f}{H^2} & 0 & -\frac{g}{f}\nabla^2 \end{pmatrix} \begin{pmatrix} \hat{q} \\ \hat{\delta} \\ \hat{a} \end{pmatrix}$$

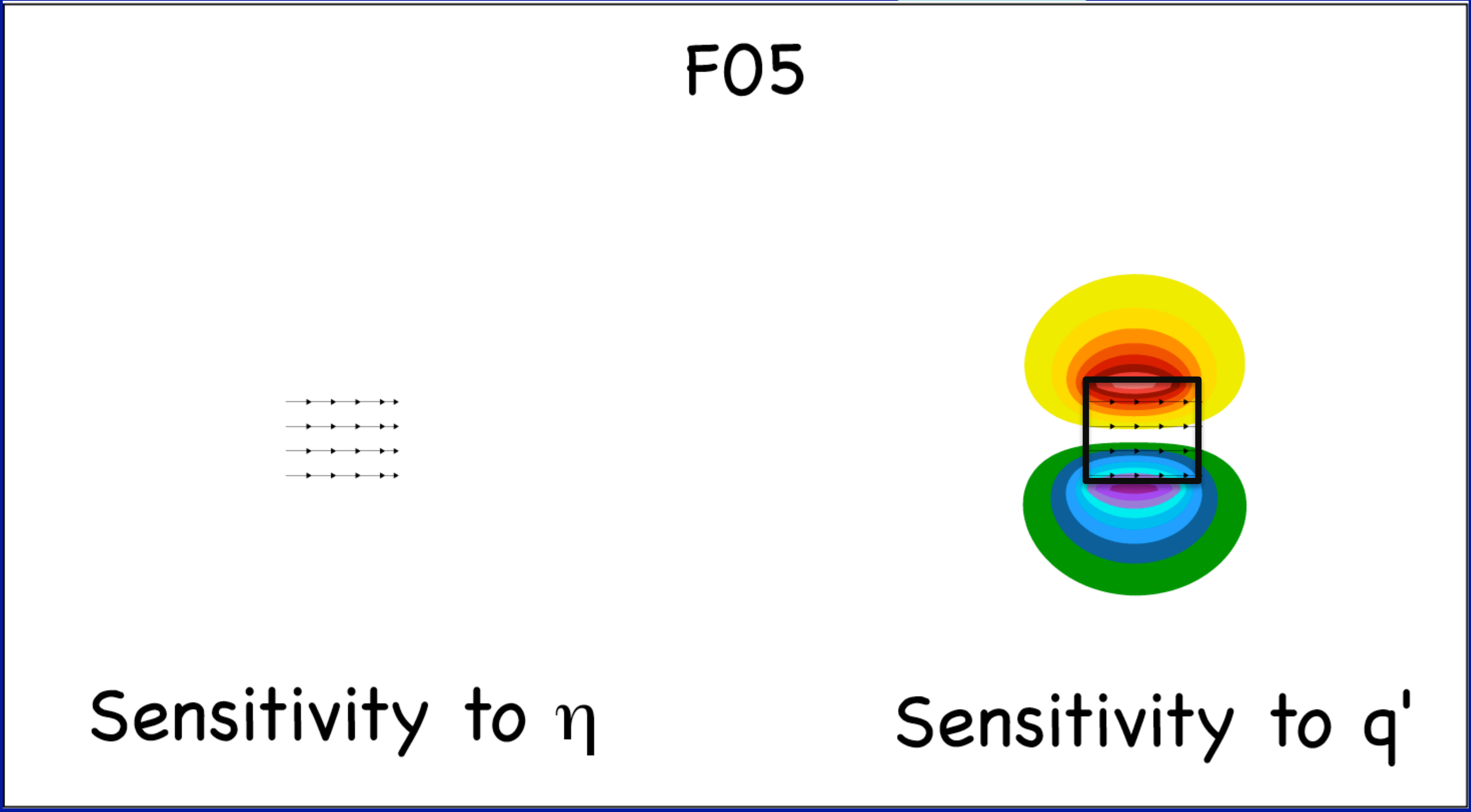
Sensitivities to *q*, *δ*, and the imbalance, *a*

$$\nabla^2 \hat{q} - \frac{f^2}{gH}\hat{q} = \frac{Hf}{g}[\hat{\eta} - \frac{g}{f}(\frac{\partial \hat{v}}{\partial x} - \frac{\partial \hat{u}}{\partial y})]$$

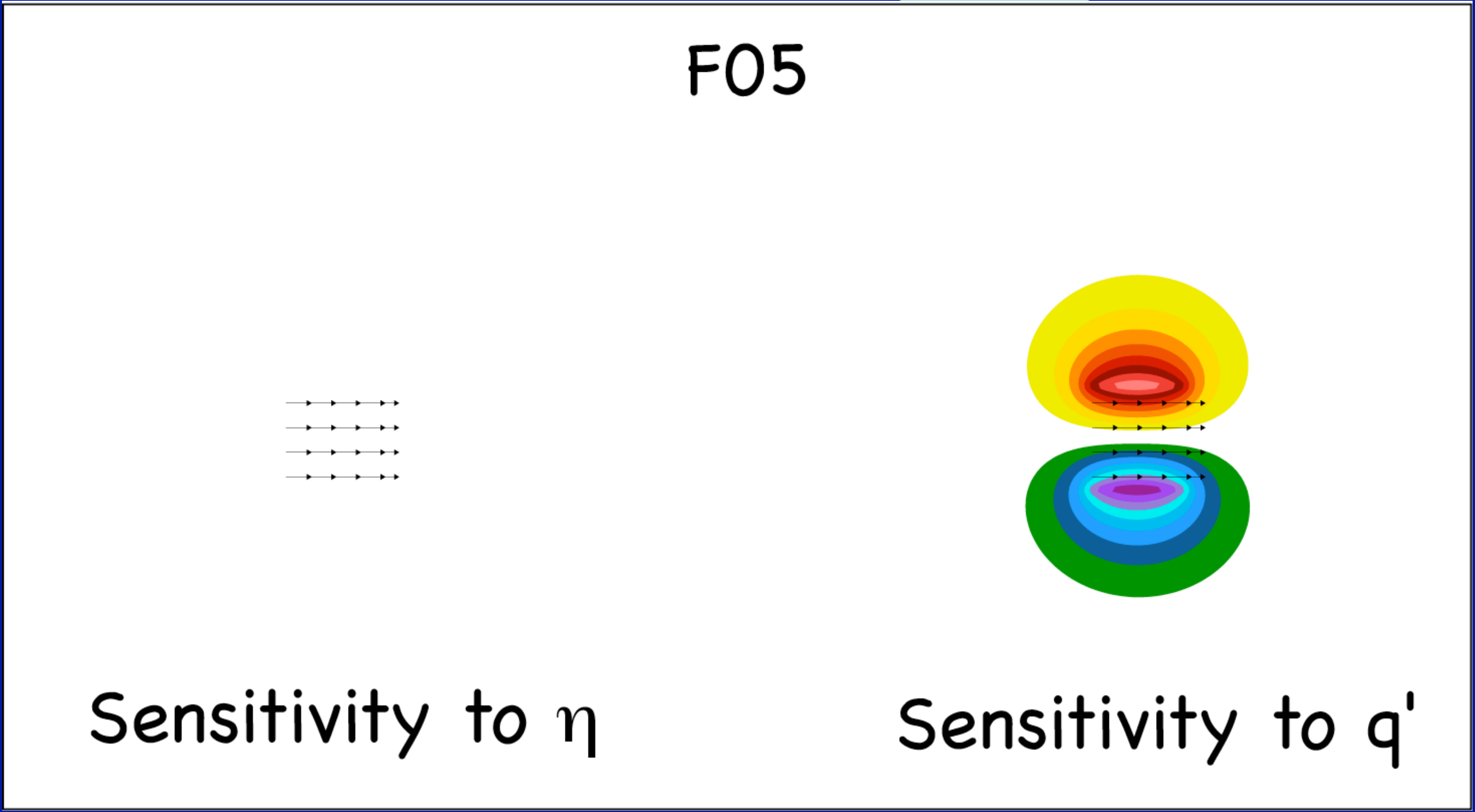
$$\nabla^2 \hat{\delta} = -(\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y})$$

$$\nabla^2 \hat{a} = \frac{\partial \hat{v}}{\partial x} - \frac{\partial \hat{u}}{\partial y} - \frac{1}{H}\nabla^2 \hat{q}$$

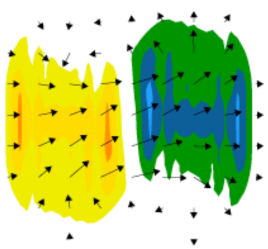
Initial condition for adjoint: $\hat{u} = 1$



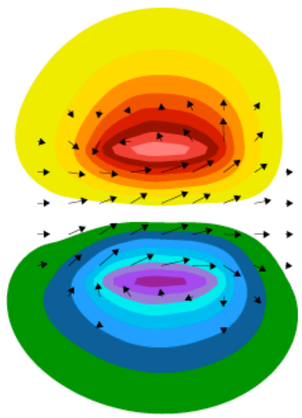
Initial condition for adjoint: $\hat{u} = 1$



F04

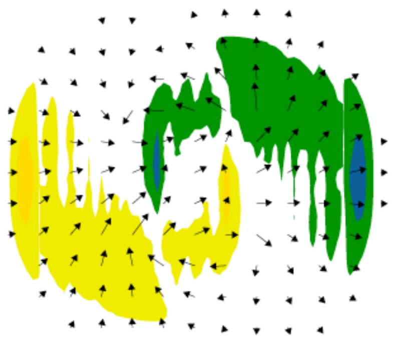


Sensitivity to η

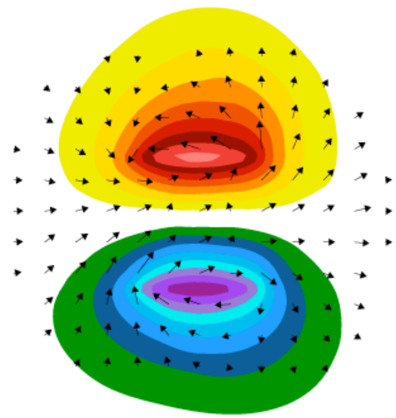


Sensitivity to q'

F03

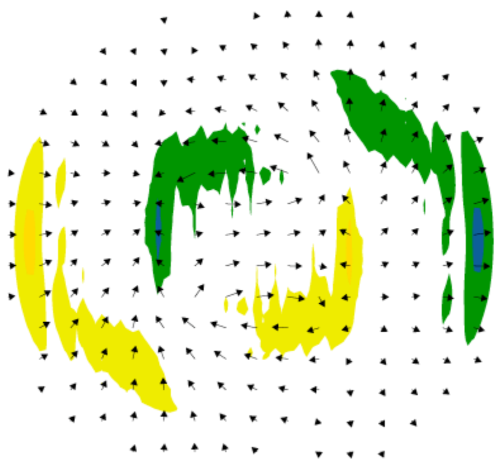


Sensitivity to η

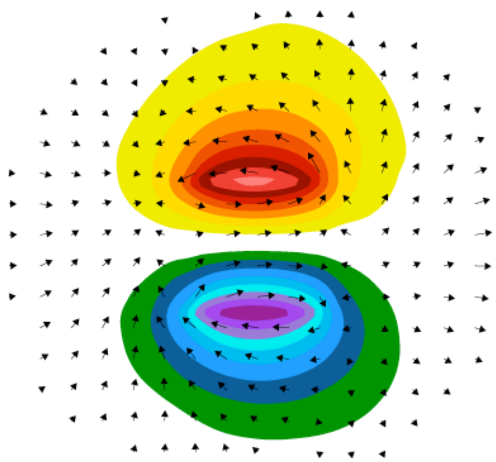


Sensitivity to q'

F02

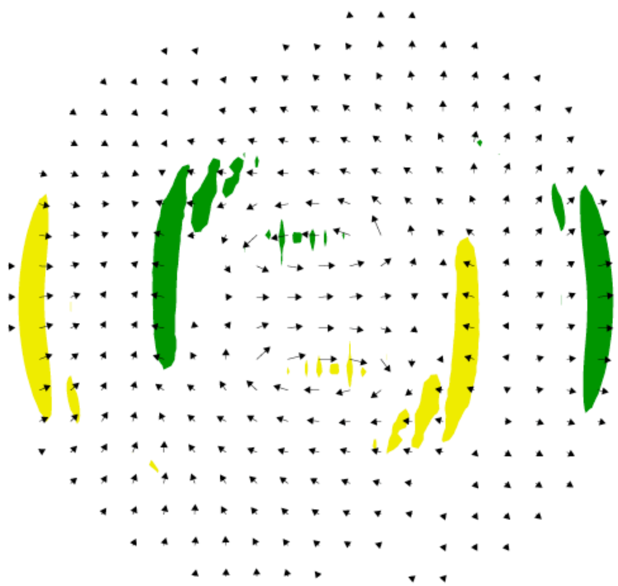


Sensitivity to η

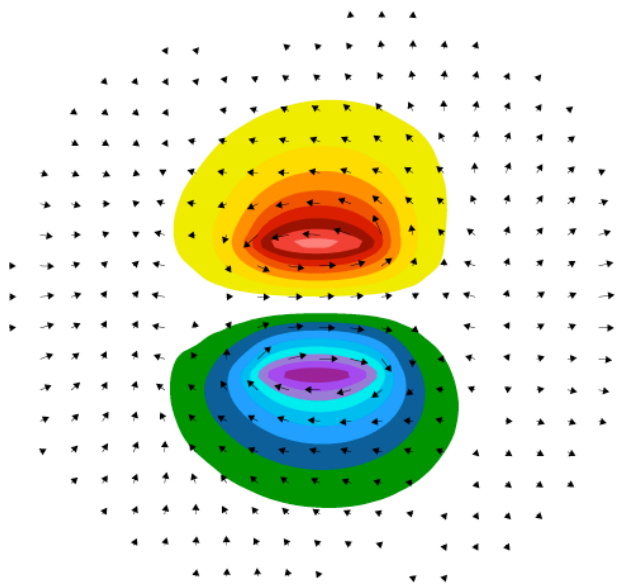


Sensitivity to q'

F01

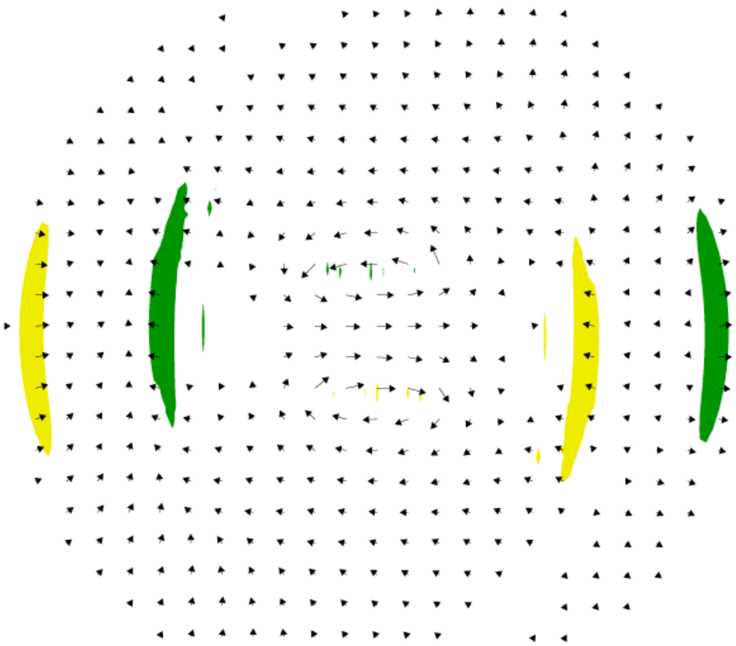


Sensitivity to η

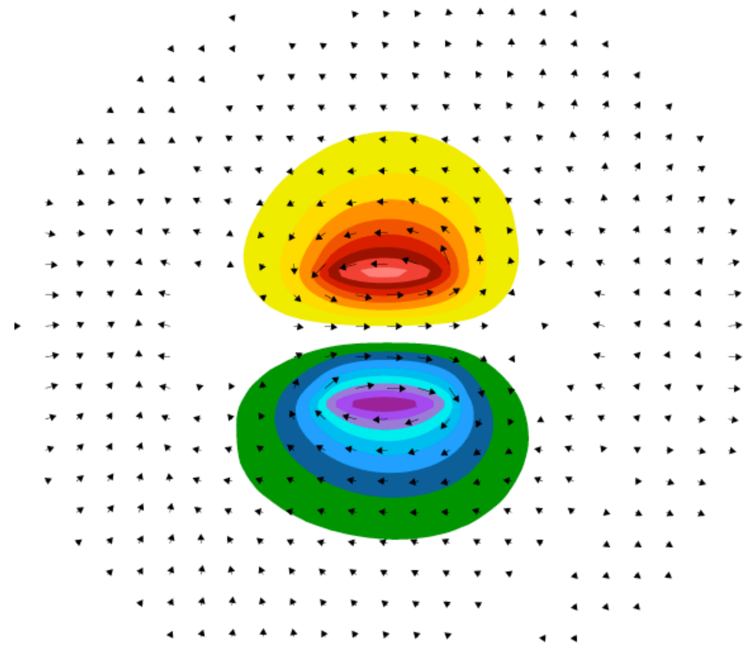


Sensitivity to q'

F00



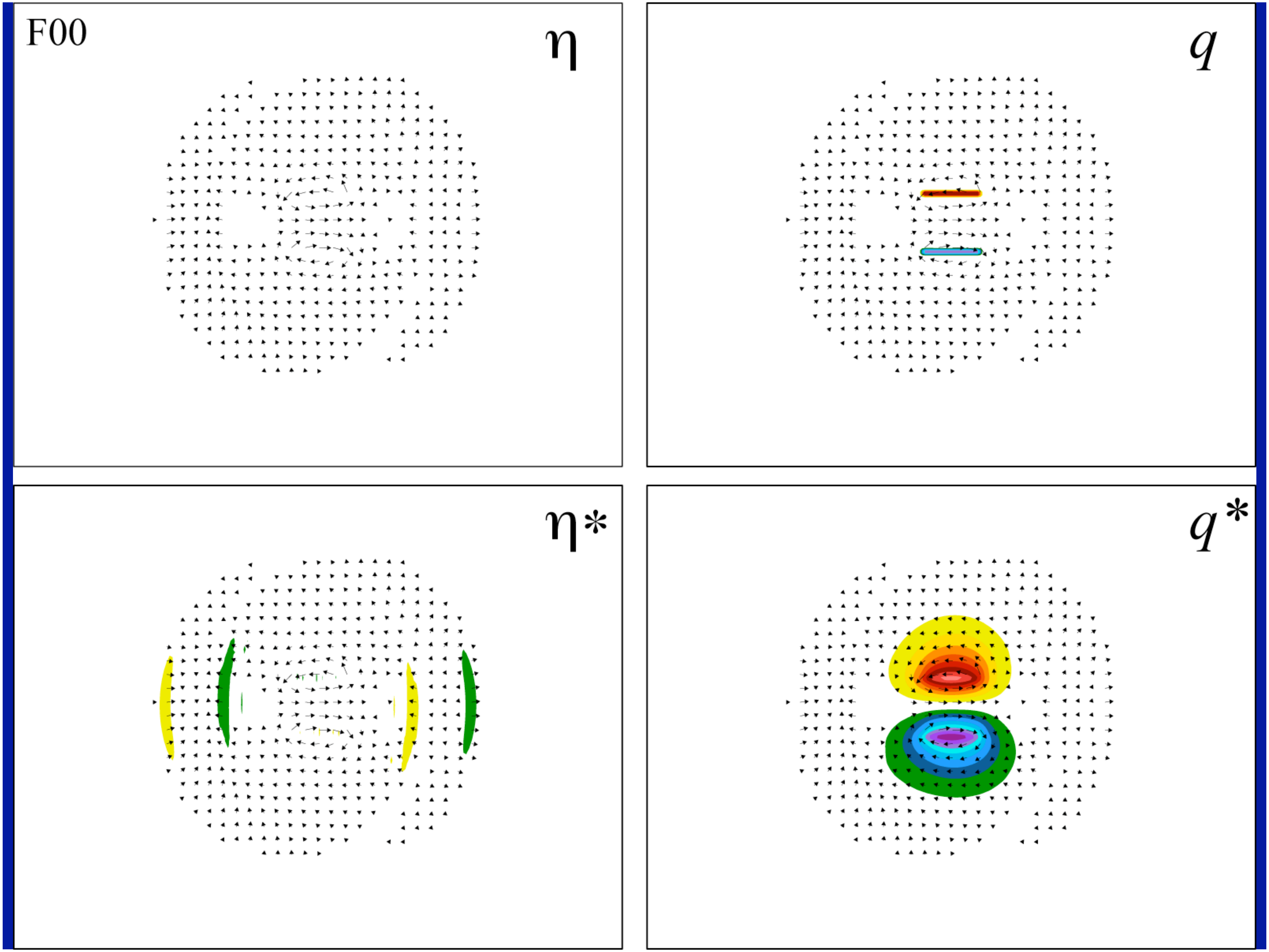
Sensitivity to η

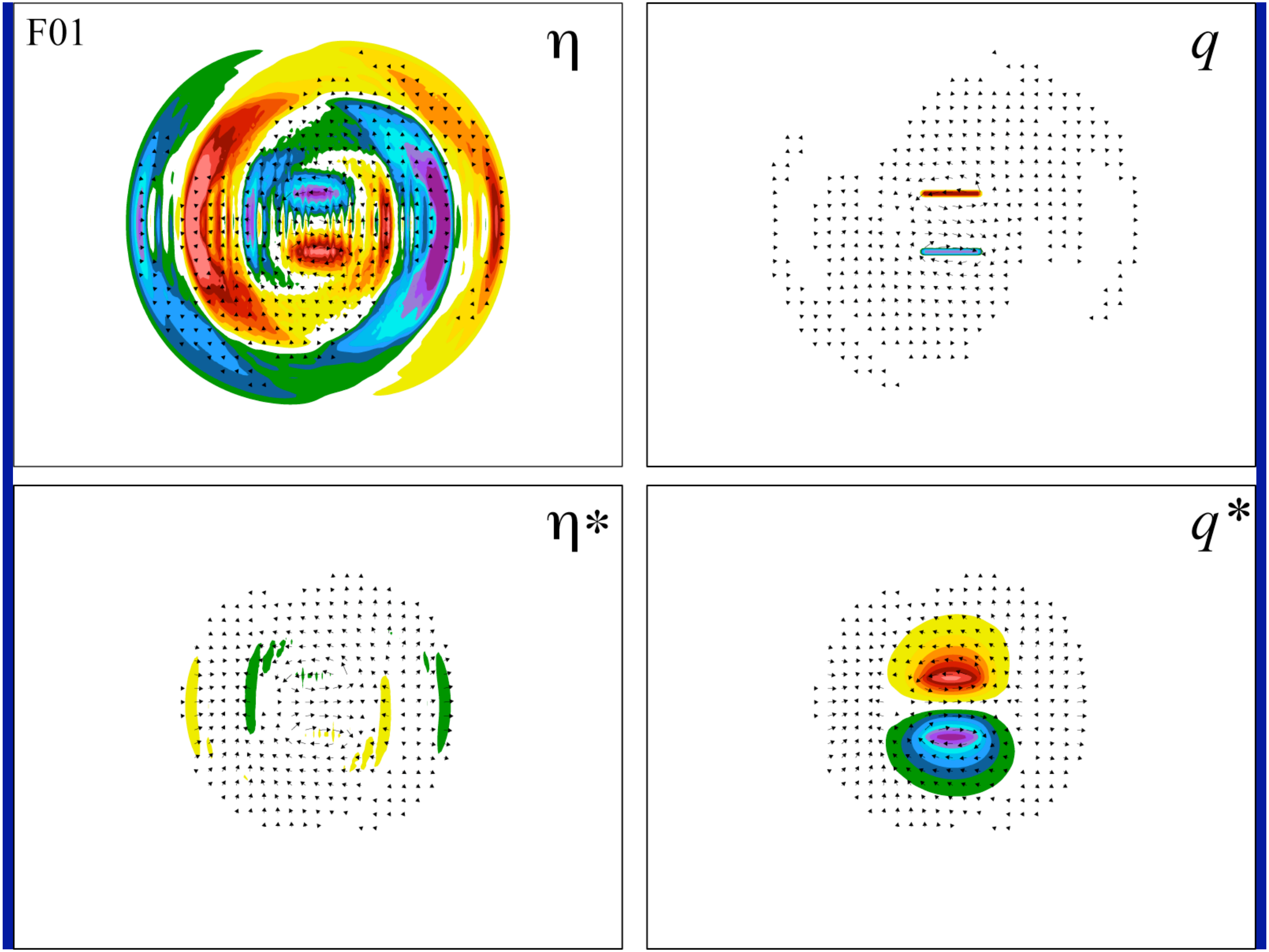


Sensitivity to q'

Geostrophic adjustment

- Forward (linearized) model integrated forward using sensitivity gradients as initial conditions . . .


F00
η
q
η*
q*


F01
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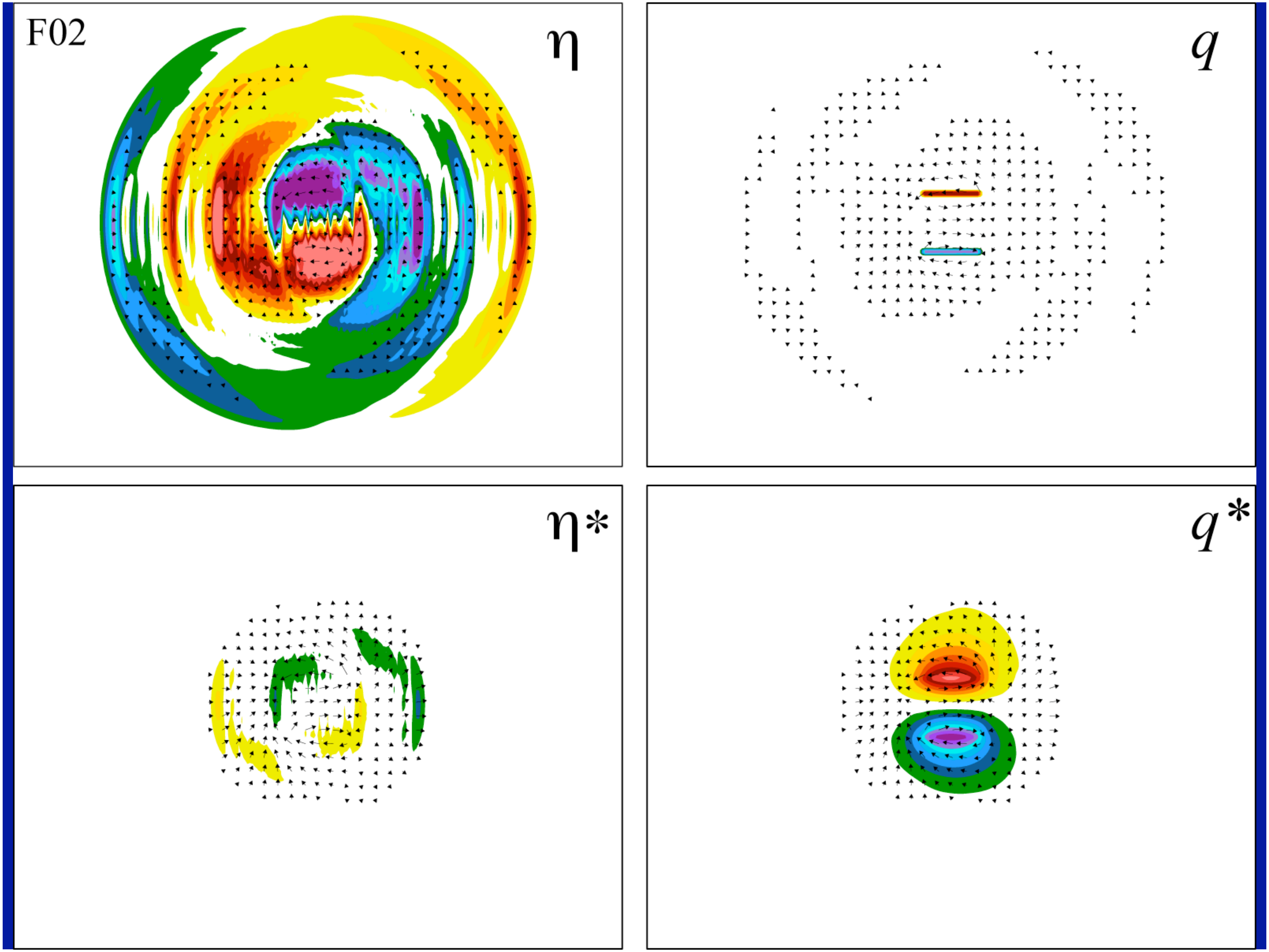
F02
η
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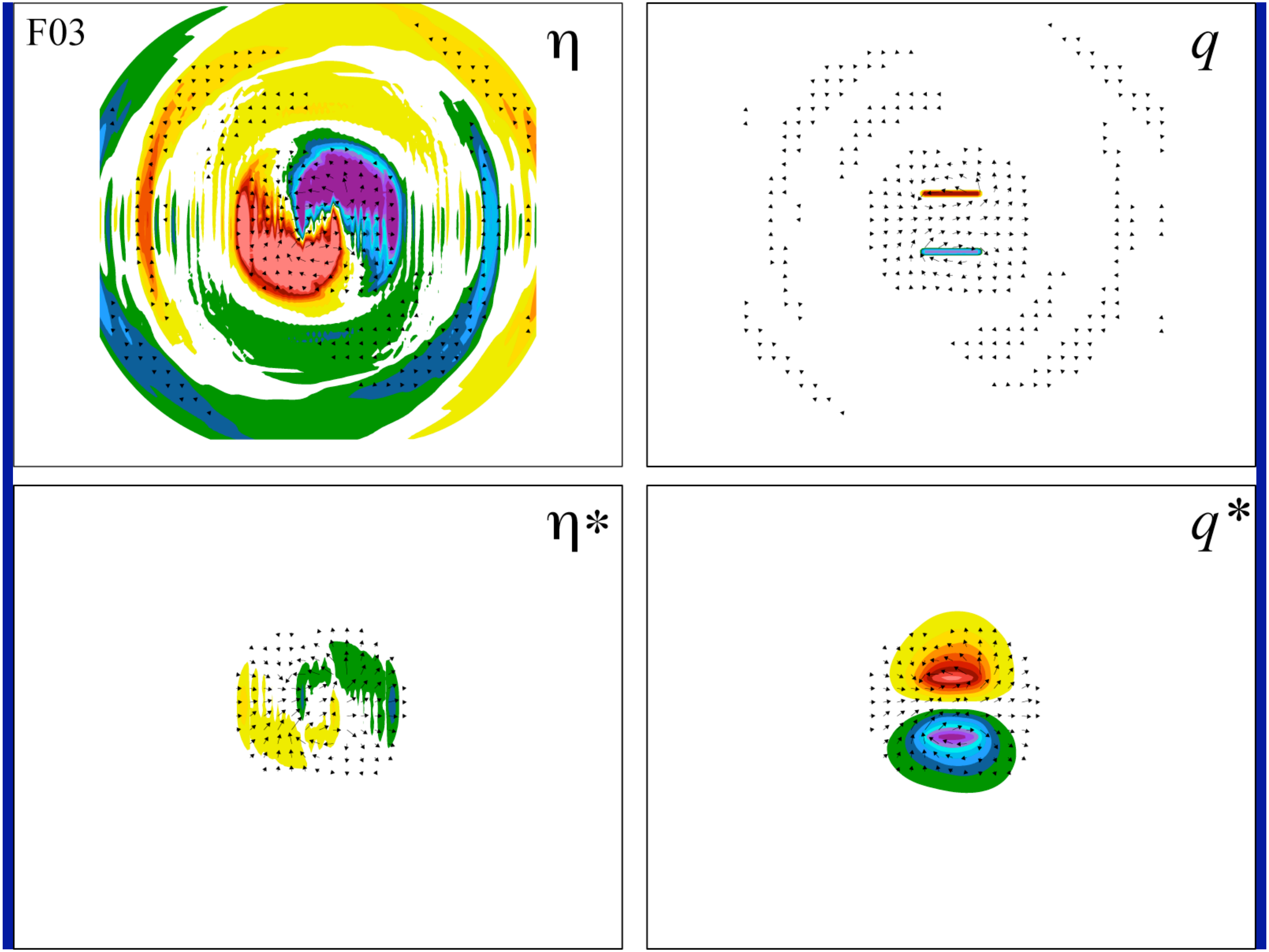
F03
η
q
η*
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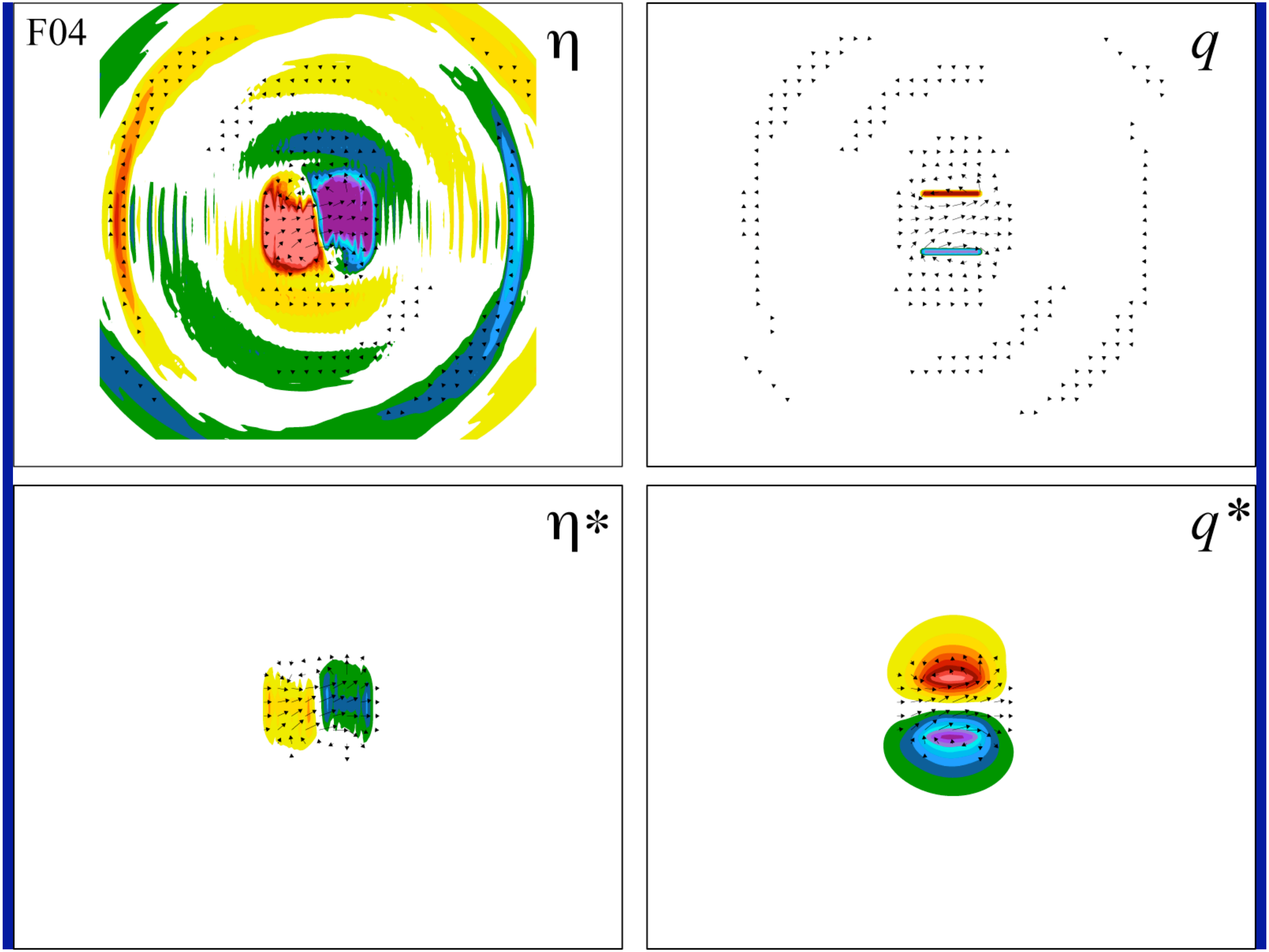
F04
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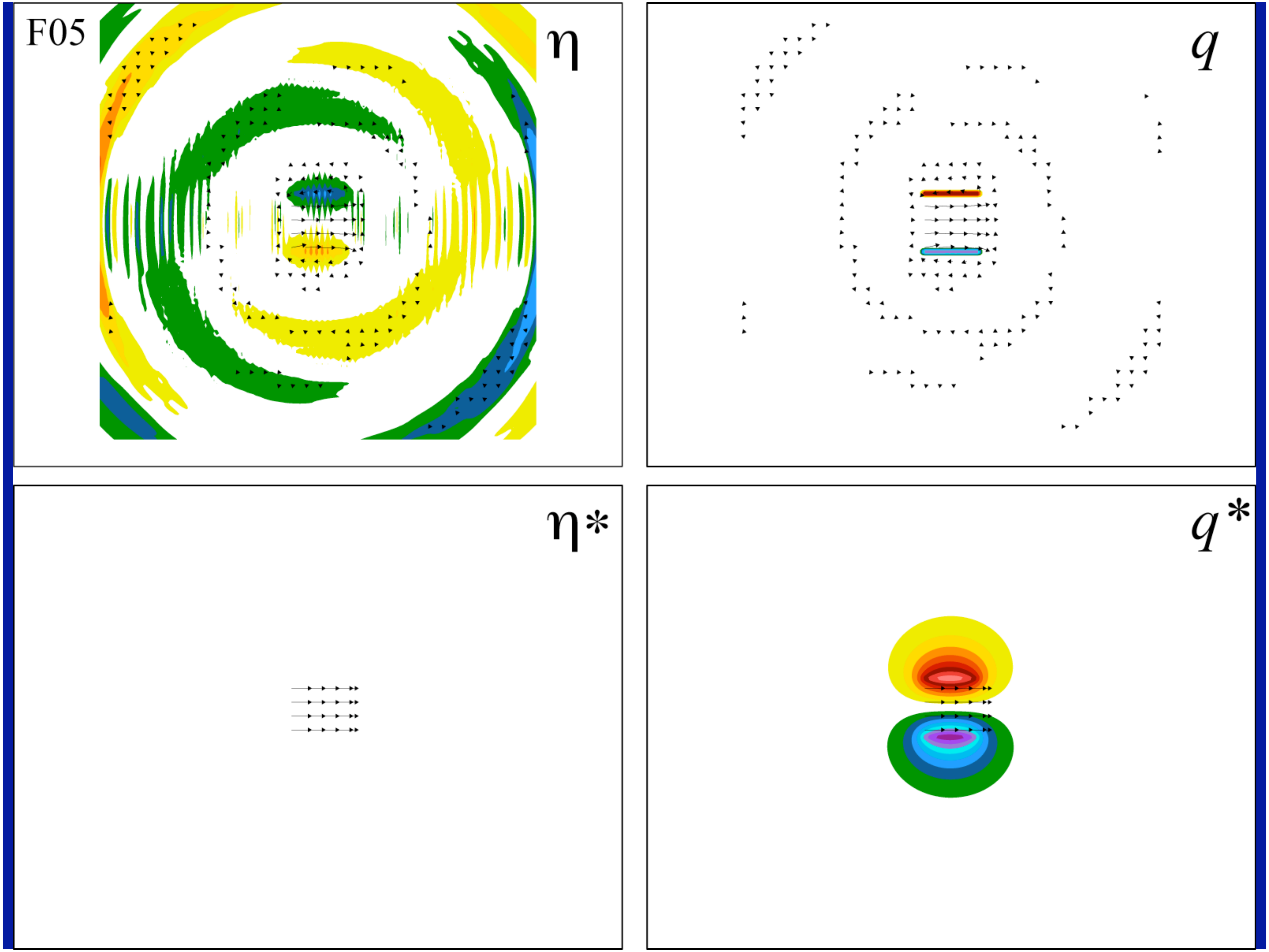
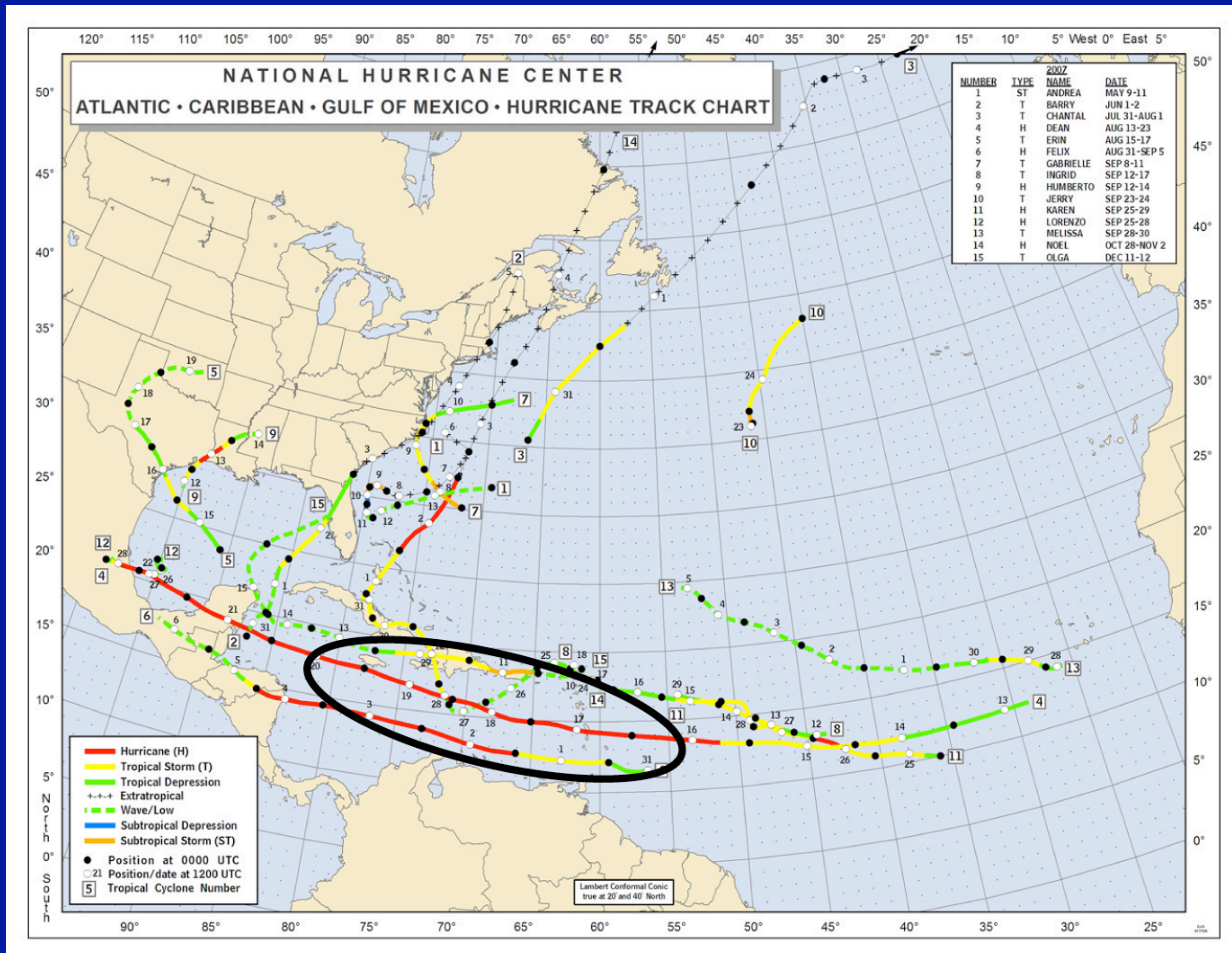

F05
η
q
η*
q*

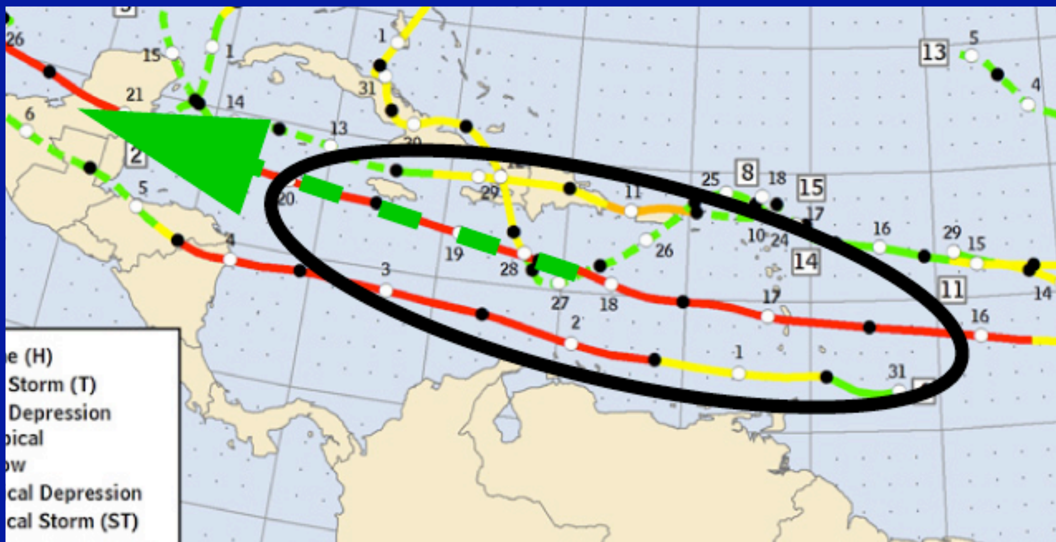
Summary (1)

- Sensitivities to PV appear to be locally "conserved" – while sensitivities to wind (vorticity) and height "evolve".
- Scale of PV sensitivity is large, while the scale of the height sensitivity is small
- Perturbations derived from the sensitivity gradients "remember" the future PV distribution associated with the response function.

Defining *R* using PV : TC steering



Hurricane Dean (2007)



At 1200 UTC 8 August 2007 Hurricane Dean was moving west-northwest at 15 kt.

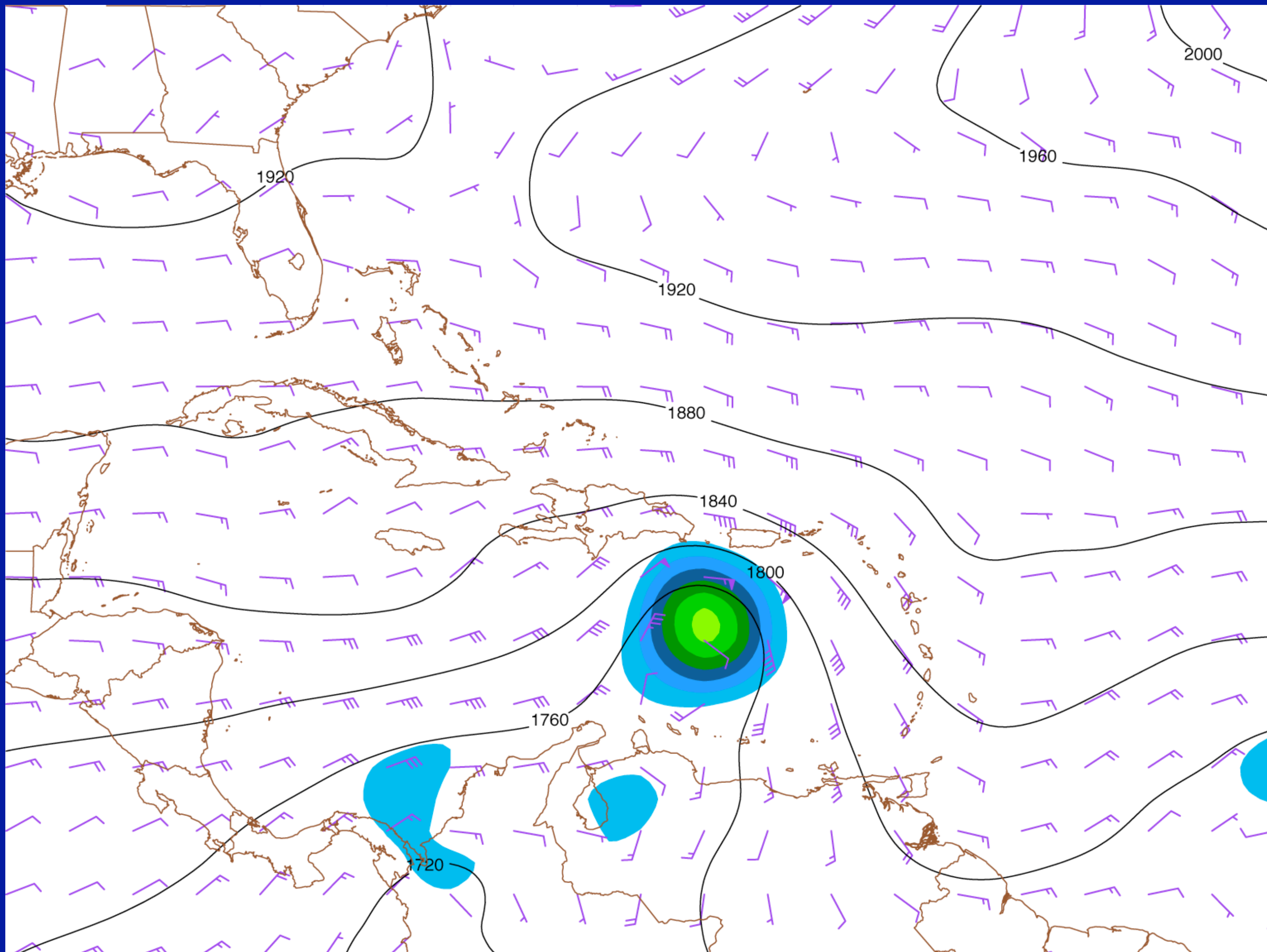
Adjoint models can be used to address the question:

For a given NWP forecast, how sensitive is tropical cyclone ***steering*** *to changes in an NWP model forecast trajectory?*

What's an appropriate response function?

How do we separate the TC from the larger-scale flow?

Streamfunction, vorticity, and wind 1200 UTC 8 August 2007



Tropical cyclone steering

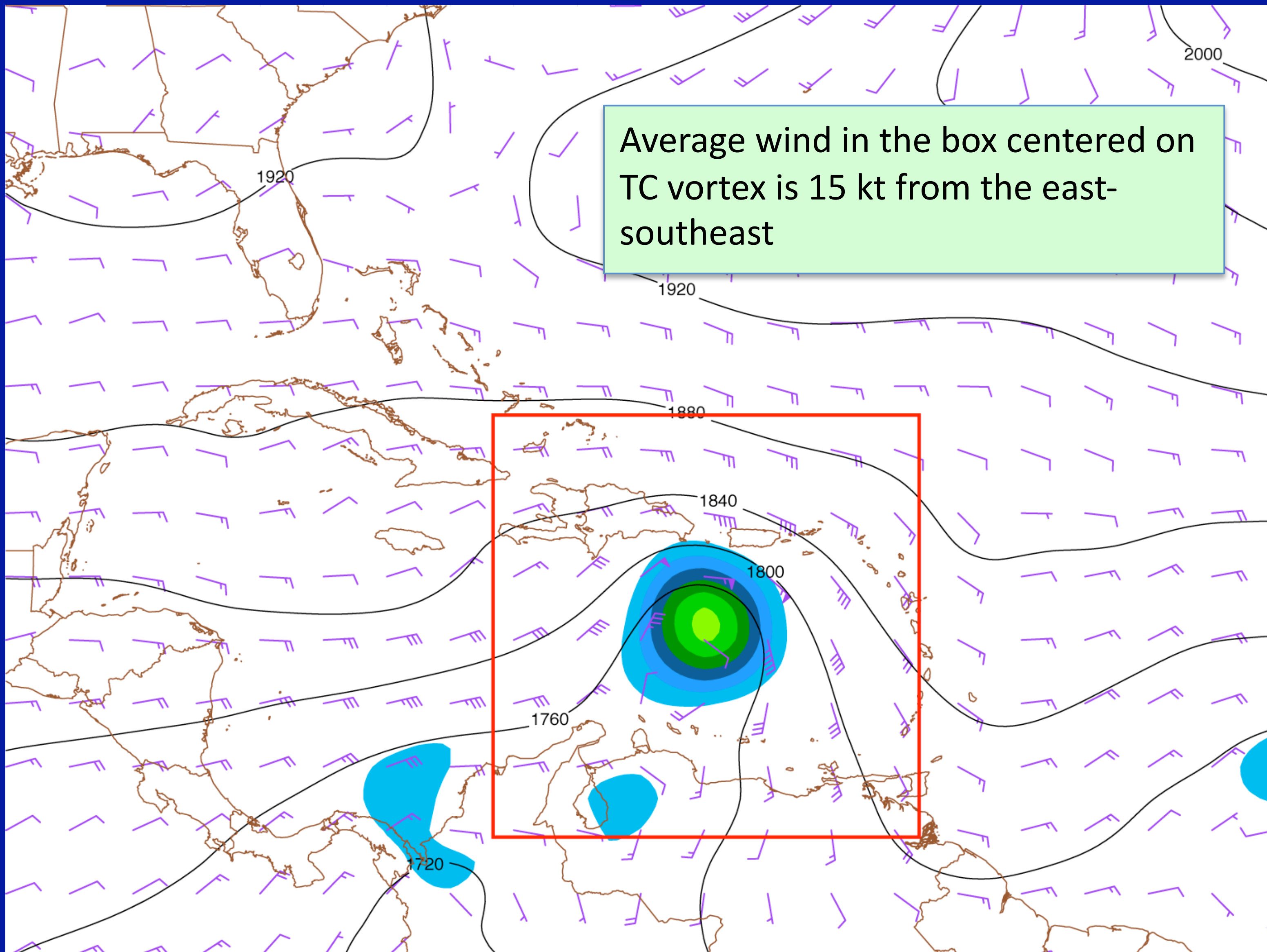
- Assuming that TC motion is largely determined by an "environmental" steering, and that this steering varies slowly over the location of the TC, the challenge is to identify that steering as the difference between the observed flow and the TC's flow.
- The horizontal flow, $\mathbf{V}$, can be decomposed as:

$$\mathbf{V} = (u,v) = (u_s + u_{\mathrm{TC}}, v_s + v_{\mathrm{TC}})$$

- Assuming that the TC flow is axisymmetric about the TC center, the averaged flow *about the center* would be the steering flow:

$$\overline{\mathbf{V}} = (\overline{u},\overline{v}) = (u_s + \overline{u}_{\mathrm{TC}}, v_s + \overline{v}_{\mathrm{TC}}) = (u_s, v_s)$$

Streamfunction, vorticity, and wind 1200 UTC 8 August 2007



A steering response function?

- Wu et al. 2006 chooses the averages of the components of the horizontal winds in a domain centered at the TC forecast position as response functions to investigate the sensitivity of TC steering to changes in the forecast trajectory:

$$R_u = \overline{u} \quad \text{and} \quad R_v = \overline{v}$$

- There are two fundamental problems with this approach
  - Perturbations to the forecast trajectory must leave the TC at the same position (and in the averaging domain)
  - No real separation of the environmental flow and disturbance (feature) flow has been made

Changes in the location and structure of the TC can change the diagnosed steering in unintended ways. (next talk!)

A (potential) vorticity based partitioning

- TCs are localized coherent vortices.
- The influence of PV is non-local.
- The strong azimuthal circulation of the TC vortex could "surgically" removed from the full flow by zeroing out the PV (vorticity) of the vortex.
- The environmental flow (the flow not associated with the TC) could be recovered by inverting the remaining PV (vorticity) to recover the winds. That environmental flow can then be averaged.

$$u, v, T \rightarrow q \rightarrow q_{env} \xrightarrow{\text{PV inversion}} u_{env}, v_{env} \rightarrow \overline{u}_{env}, \overline{v}_{env}$$

Defining a steering response function

$$\mathbf{x}_f \rightarrow \mathbf{Q}(\mathbf{x}_f) \rightarrow \mathbf{EQ}(\mathbf{x}_f) \rightarrow \mathbf{Q}^{-1}(\mathbf{EQ}(\mathbf{x}_f)) \rightarrow \mathbf{x}_f^{env} \rightarrow R(\mathbf{x}_f^{env})$$

$$\frac{\partial R}{\partial \mathbf{x}_f} \leftarrow \mathbf{Q}^{adj}\mathbf{E}^{adj}(\mathbf{Q}^{-1})^{adj}\frac{\partial R}{\partial \mathbf{x}_f^{env}} \leftarrow \mathbf{E}^{adj}(\mathbf{Q}^{-1})^{adj}\frac{\partial R}{\partial \mathbf{x}_f^{env}} \leftarrow (\mathbf{Q}^{-1})^{adj}\frac{\partial R}{\partial \mathbf{x}_f^{env}} \leftarrow \frac{\partial R}{\partial \mathbf{x}_f^{env}}$$

$$\frac{\partial R}{\partial \mathbf{x}_f} \leftarrow \frac{\partial R}{\partial Q_f} \leftarrow \mathbf{E}^{adj}\frac{\partial R}{\partial Q_f^{env}} \leftarrow \frac{\partial R}{\partial Q_f^{env}} \leftarrow \frac{\partial R}{\partial \mathbf{x}_f^{env}}$$

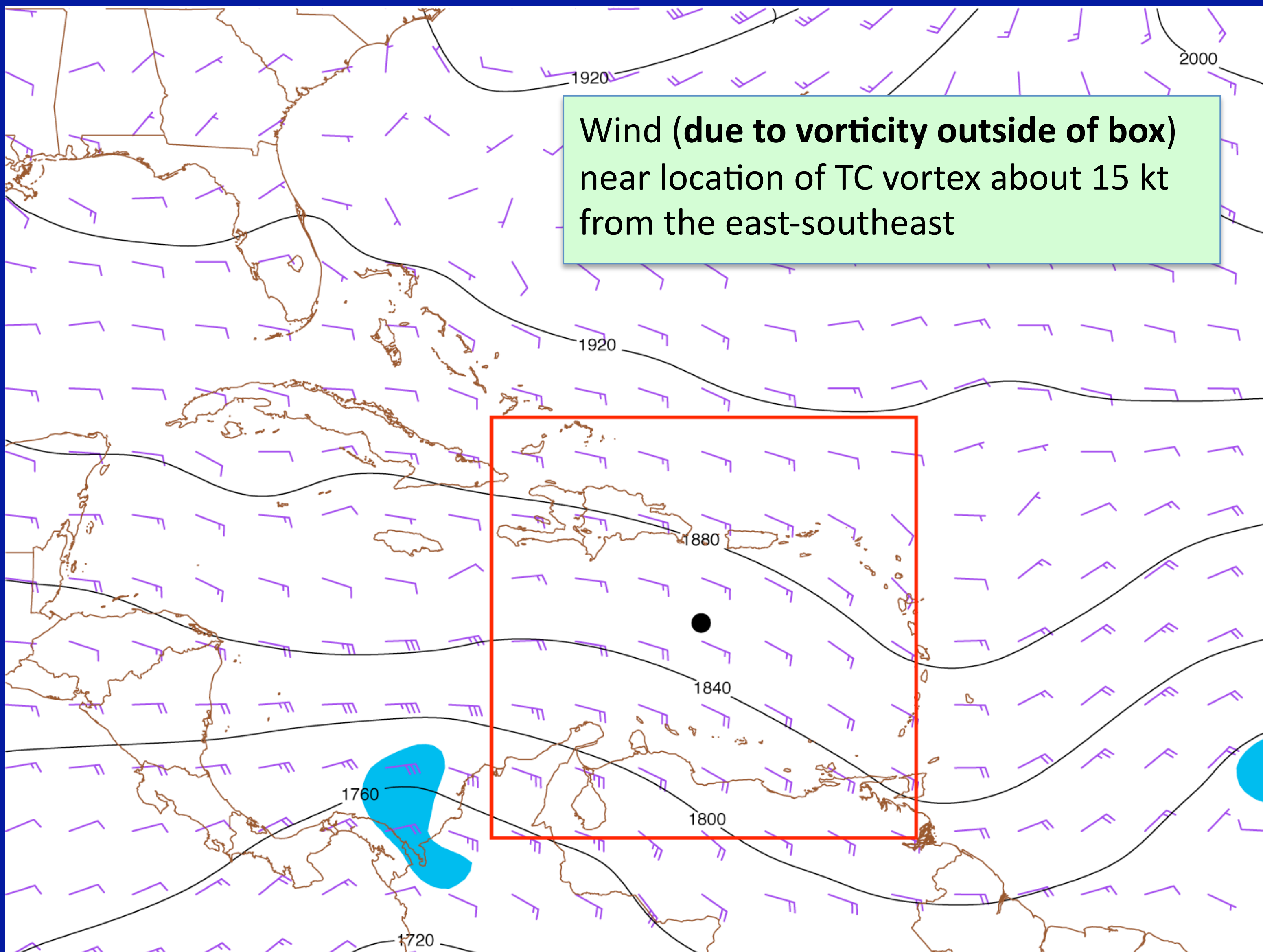
E is an environmental projection operator:

$$E = \begin{cases} 0, \text{ within disturbance domain} \\ 1, \text{ otherwise} \end{cases}$$

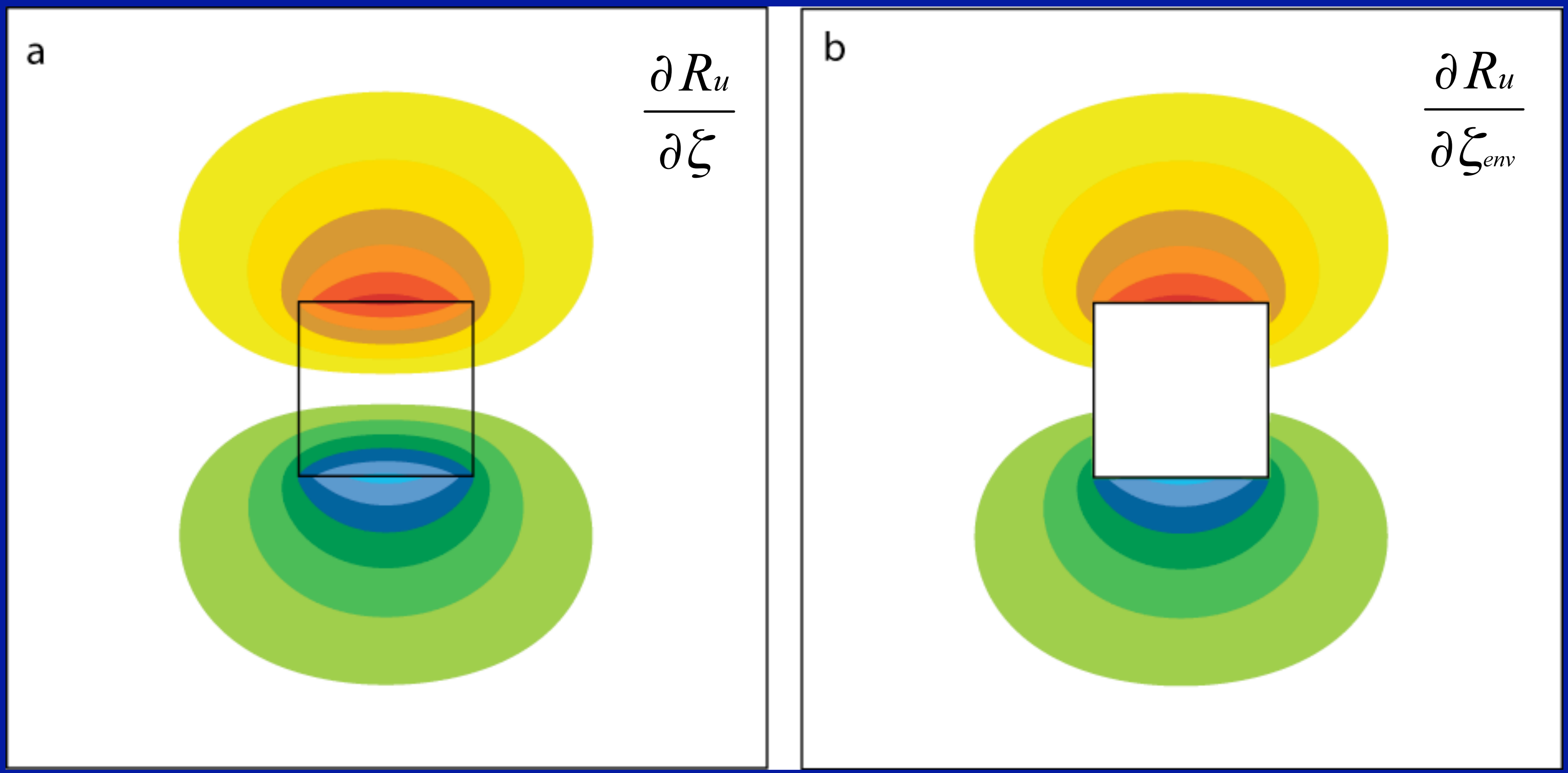
Q is a linearized PV operator

R, would be the average of environmental u and v

Vortex removed from box

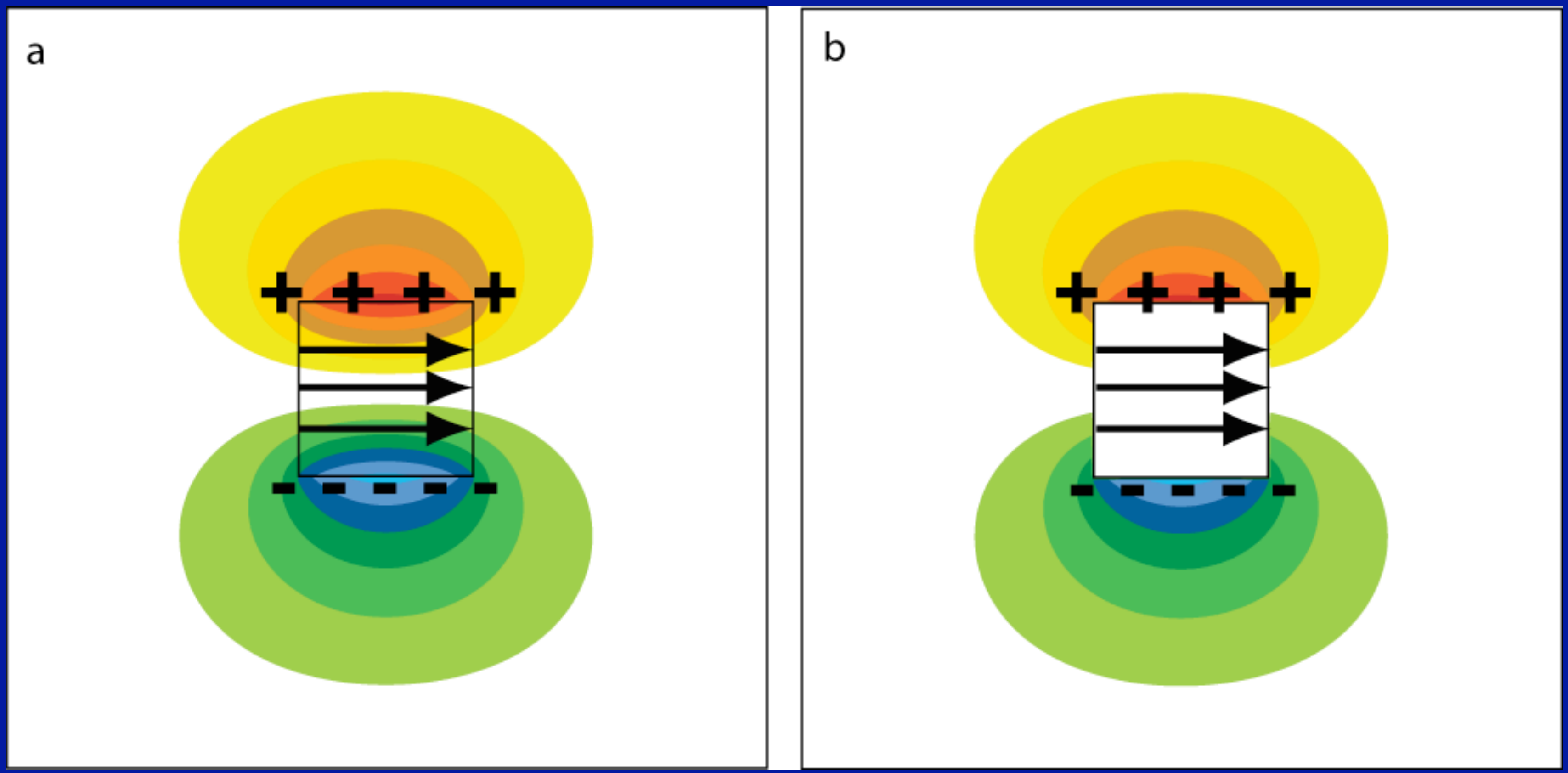


Example: Zonal wind response function



Note: TC needn't be in the center of the response function box on the right for this response function to yield the desired result – it should just be in the box!

Example: Response function for zonal flow



Summary (2)

- Interpretations of sensitivities of response functions used for the steering of tropical cyclones (TCs) suffer from a fundamental flaw – they do not measure TC steering, but rather sensitivities to the asymmetric flow associated with changes in TC position and structure.
- Using the concept of potential vorticity inversion, a more robust definition of a TC steering response function has been proposed.

Summary (2)

- More broadly, the use of environmental and local projection operators, defined using the adjoint of potential vorticity operators, offers a potentially less ambiguous separation between disturbance and environment.

Kinetic energy in local domain – has the cyclone intensified or just the gradient?