Initial perturbations for ensemble forecasting – singular vectors vs. breeding vectors

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Linus Magnusson
Stockholm University

Acknowledgements: Erland Källén; Jonas Nycander; Martin Leutbecher; SMHI

Desirable properties of initial perturbations

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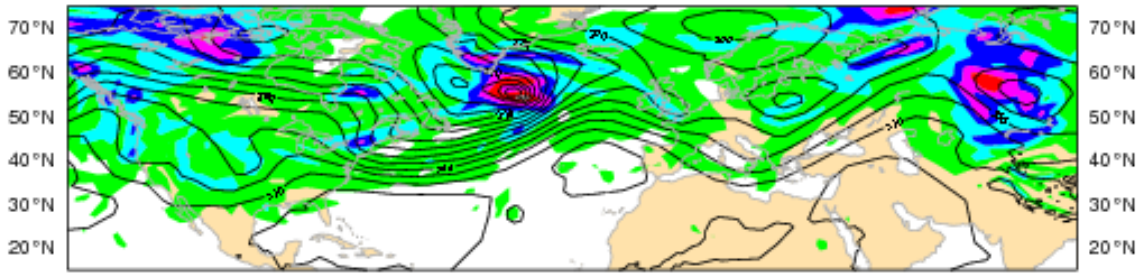
- Sampling analysis uncertainty
  - Mean amplitude
  - Geographical distribution
  - Spatial scale
  - ”Error of the day”
- Sustainable growth
- High quality of probabilistic forecast



Singular vectors

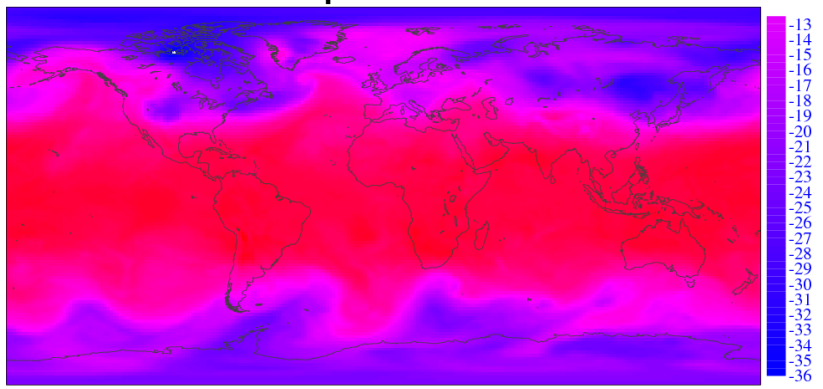
$$\alpha = \frac{\left\langle \Delta\vec{x}(t), \Delta\vec{x}(t) \right\rangle}{\left\langle \Delta\vec{x}(0), \Delta\vec{x}(0) \right\rangle}$$

Optimize perturbation growth for a time interval

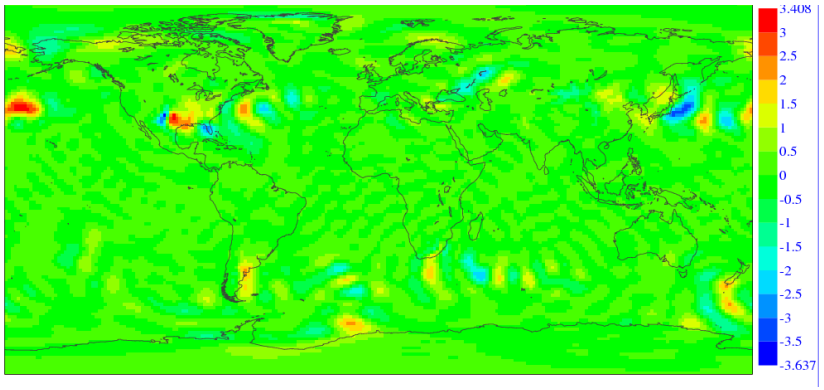
Norm dependent!

M tangent linear operator. $\Delta\mathbf{x}(t)=\mathbf{M}(t,x_0)\Delta\mathbf{x}(0)$

Atmospheric state



Singular vector perturbation



Breeding perturbations

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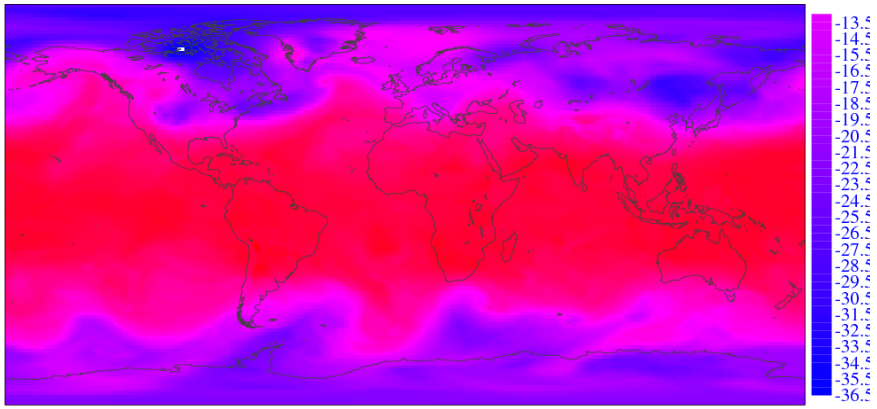
Perturbed Forecast +06h



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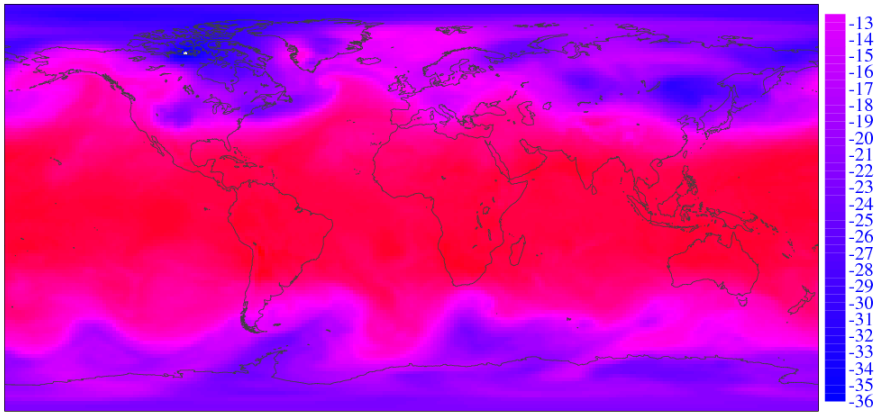
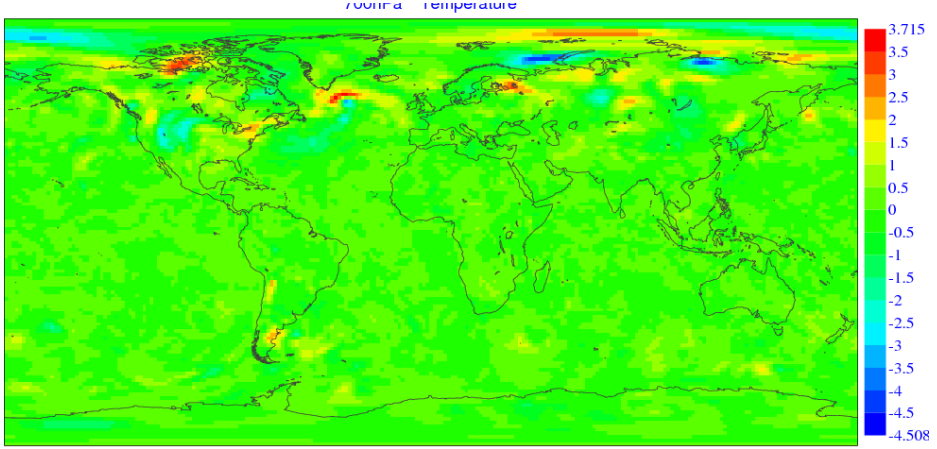
Unperturbed Forecast +06h



x normalizing factor =

Breeding vector



Random Perturbations

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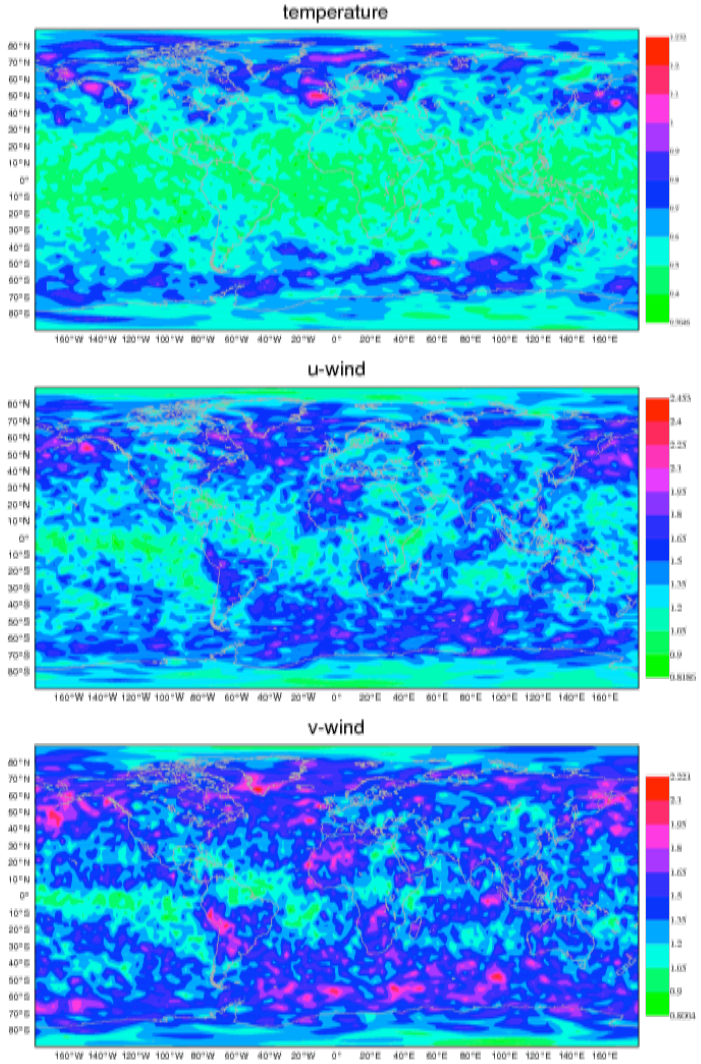
Analysis error estimate

Random number (-1 to 1) x



x global tuning factor

Random Field (RF) perturbation

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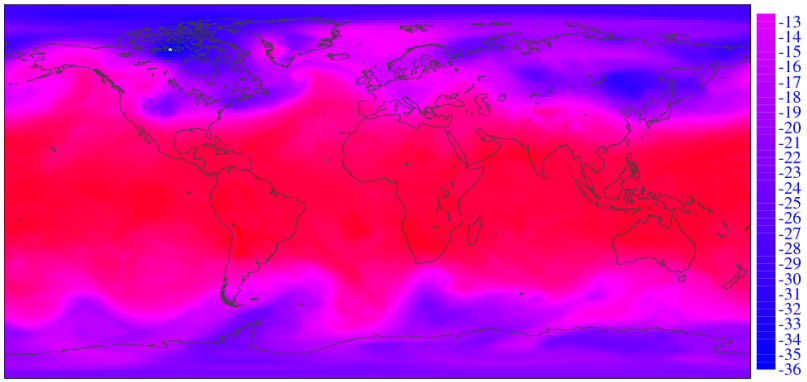
Analysis Random date 1



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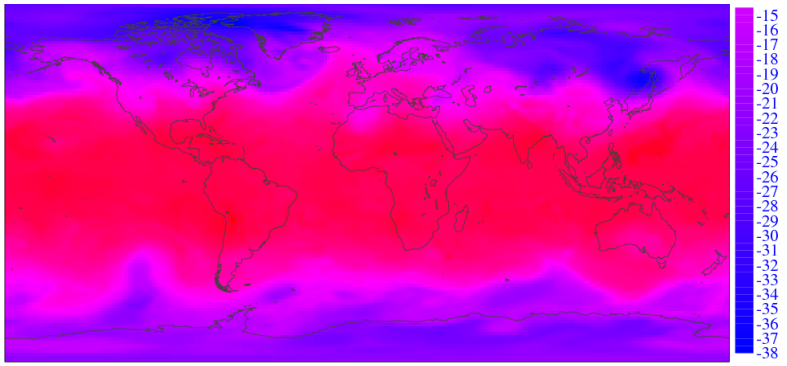
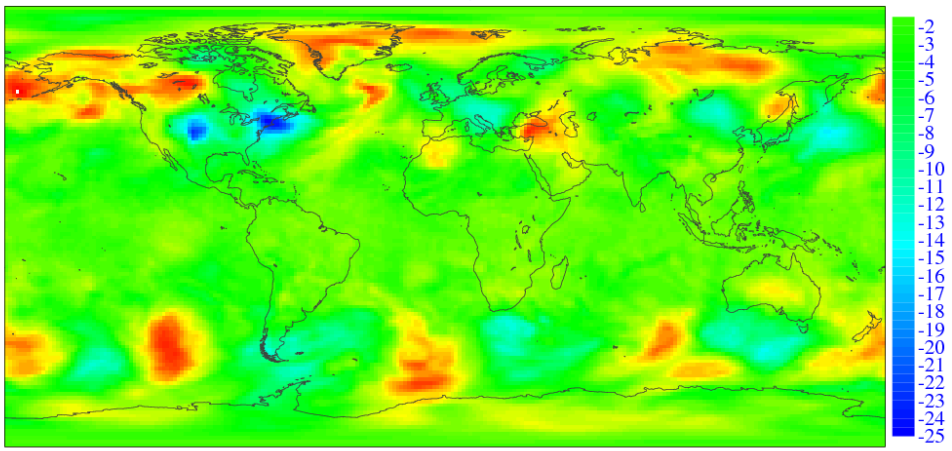
Analysis Random date 2

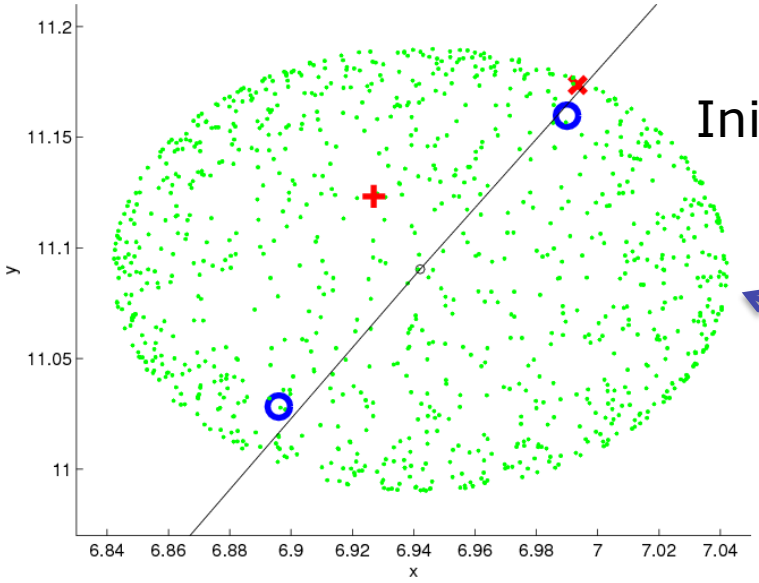


x normalizing factor =

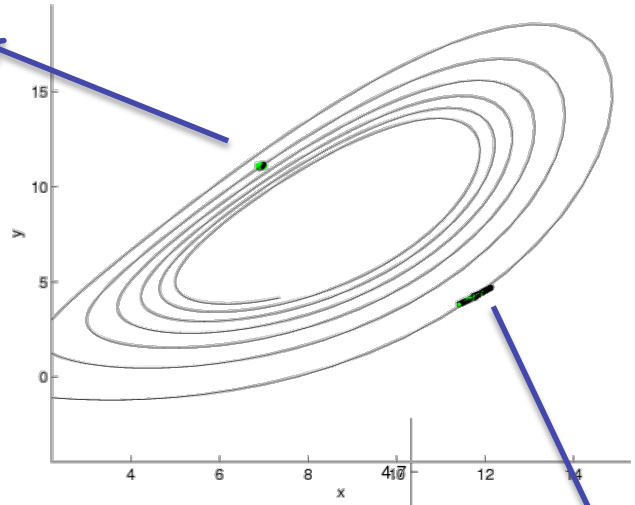
Random Field Perturbation



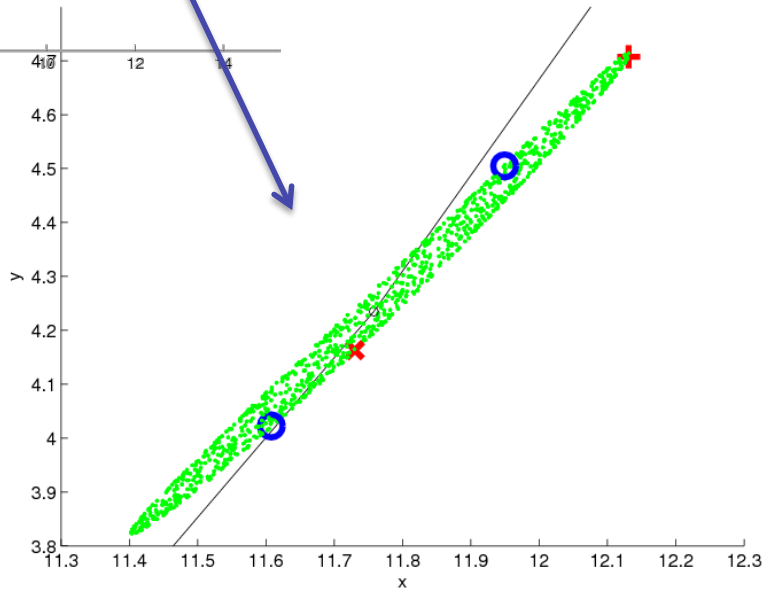
Perturbation methods
Lorenz-63



Initial point



After 1 time unit



- Random pert.
- 1st SV
- 2nd SV
- BV

Exponential perturbation growth

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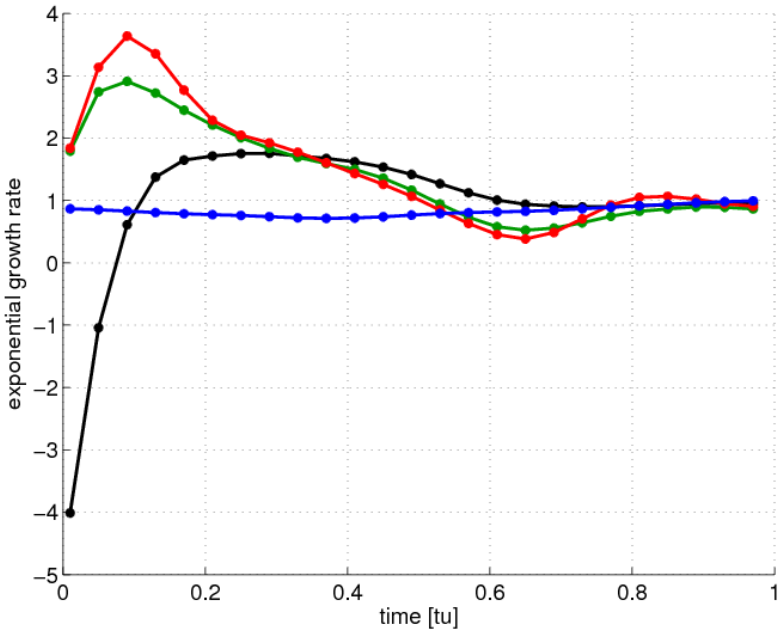
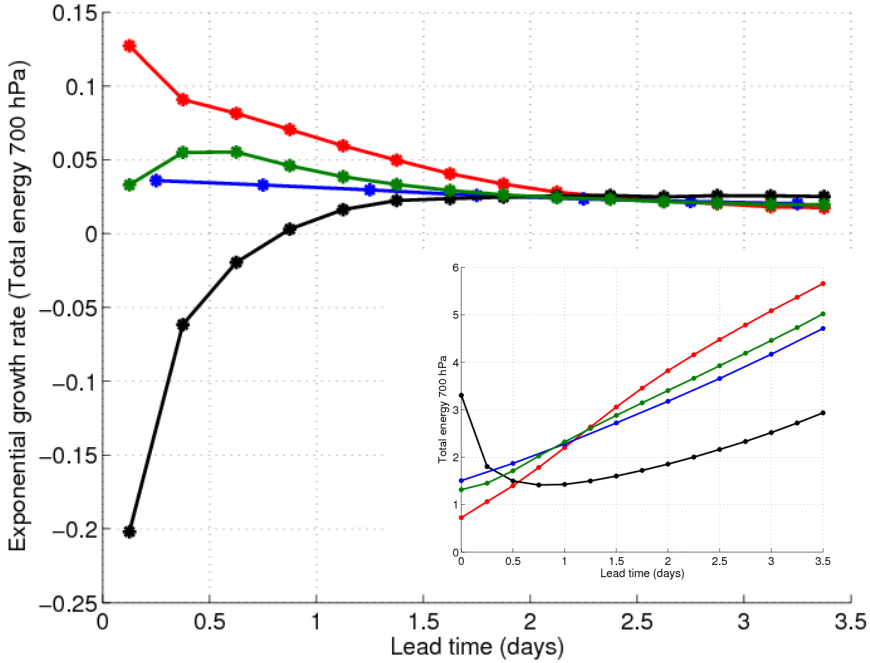
$$\lambda = \frac{1}{\Delta t} \ln \left(\frac{||\Delta x(t + \Delta t)||}{||\Delta x(t)||} \right)$$

Lorenz-63



NWP-model (ECMWF)



SV – red, BV – blue, Random Field Pert. – Green, Random Pert. - black

Evolution of ensemble spread (one case, total pert. energy 700 hPa)

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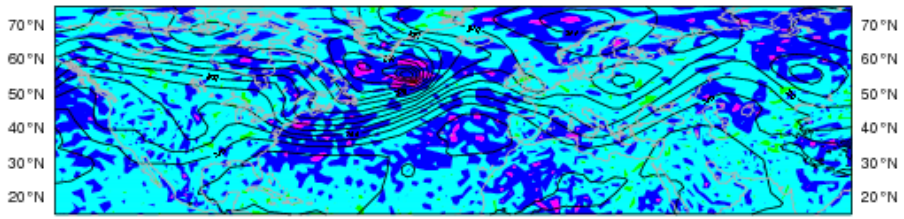
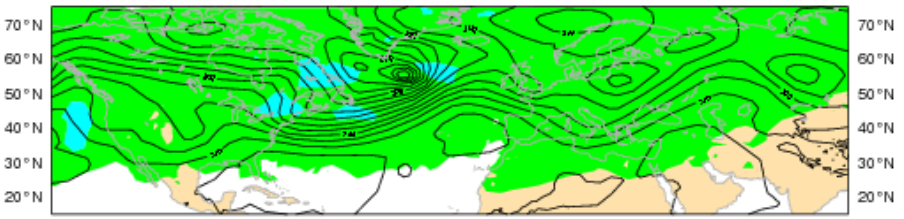
Initially

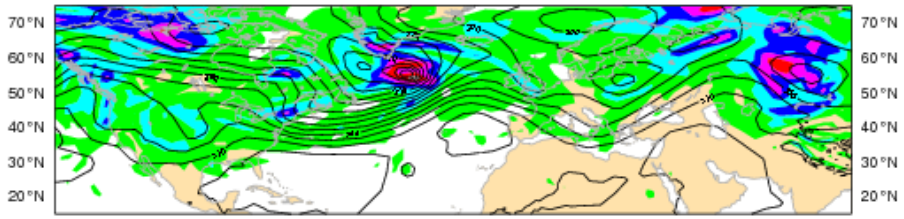
Random perturbations



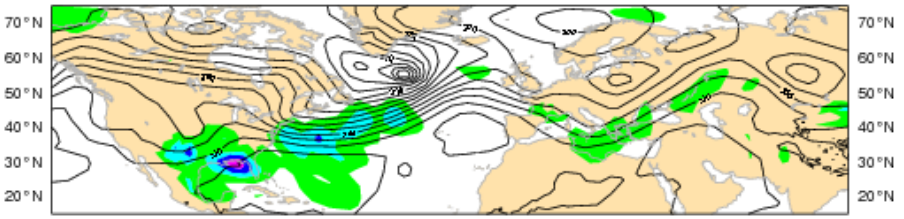
Random Field perturbations



Breeding perturbations



Singular Vector perturbations



Maximum – red,

Evolution of ensemble spread (one case, total pert. energy 700 hPa)

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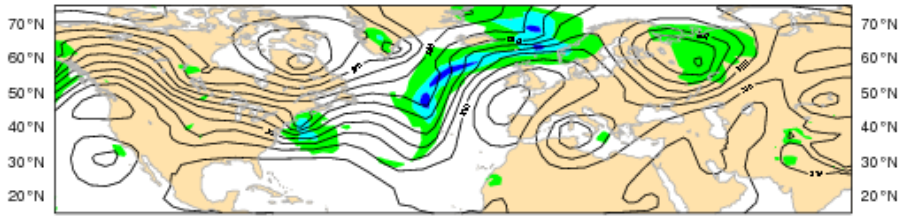
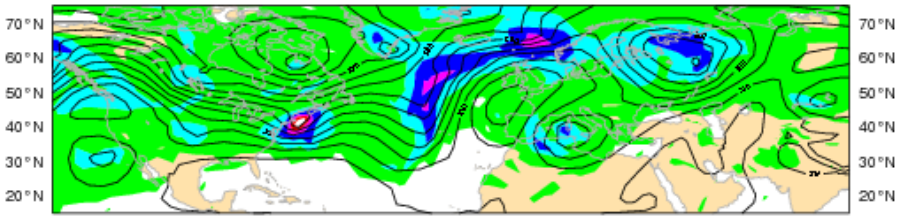
+48h

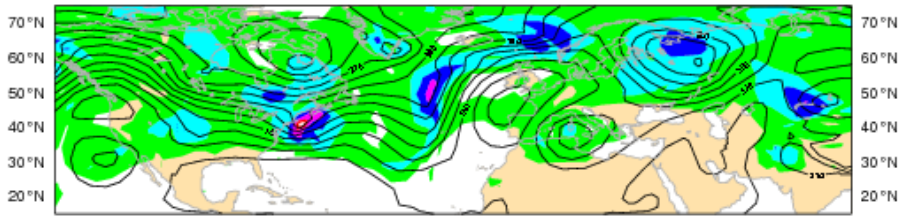
Random perturbations



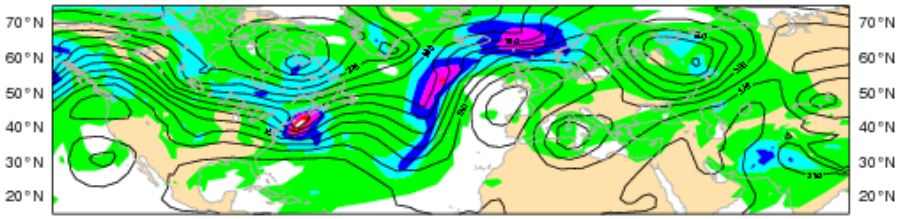
Random Field perturbations



Breeding perturbations



Singular Vector perturbations



Maximum – red, Scale 48: twice the scale for +00h

Connections between perturbations and baroclinic zones

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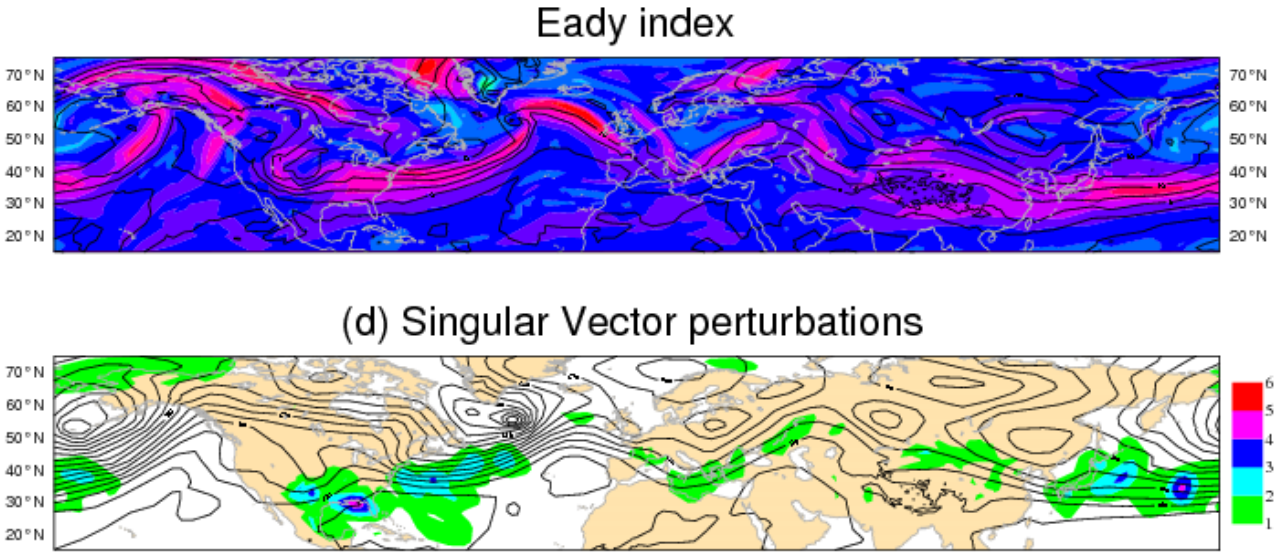
$$E = 0.3125 \frac{f}{N} \frac{dV}{dz}$$ *Fastest growth rate of normal modes*



Correlation Eady index – Ens. Stdev z500

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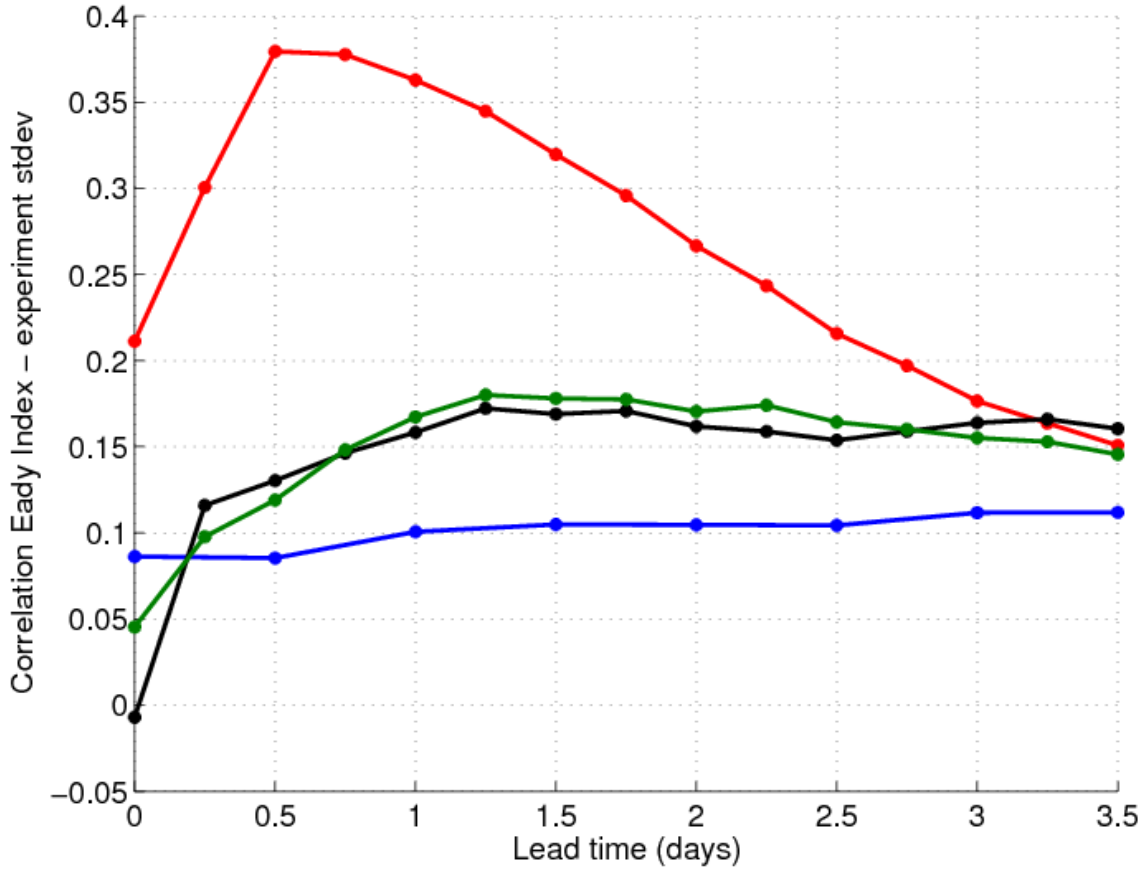

SV - Red
ET - Blue
RF – Green
RP - Black

(20N-70N)

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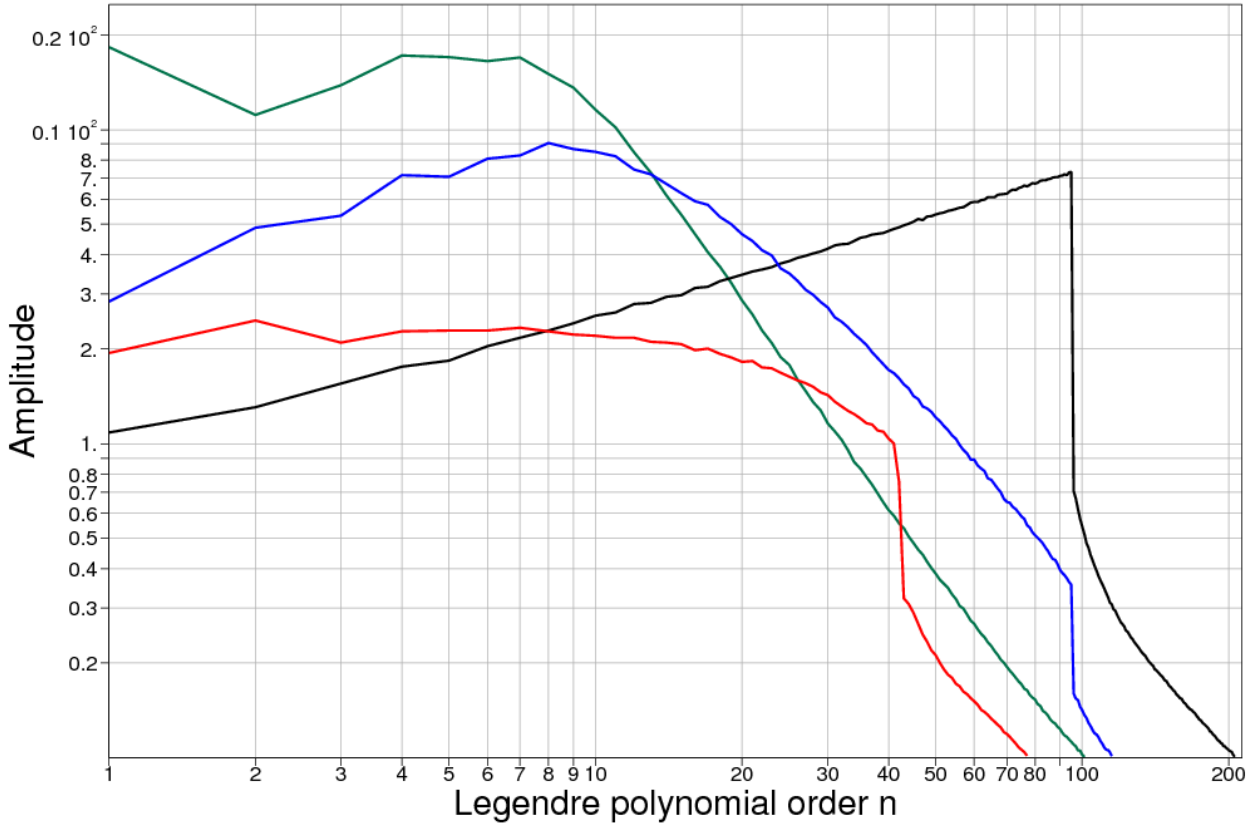
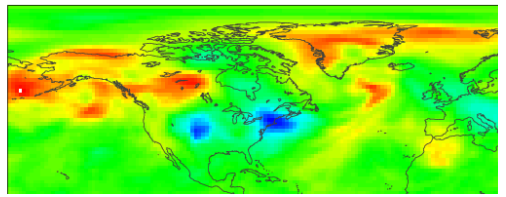
Perturbation amplitude spectra T700 +00

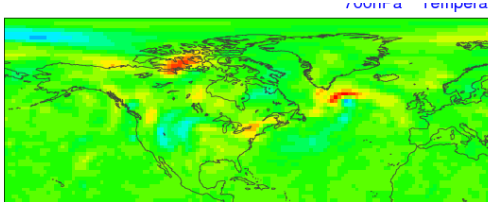


SV - Red
ET - Blue
RF – Green
RP - Black

RF



BV



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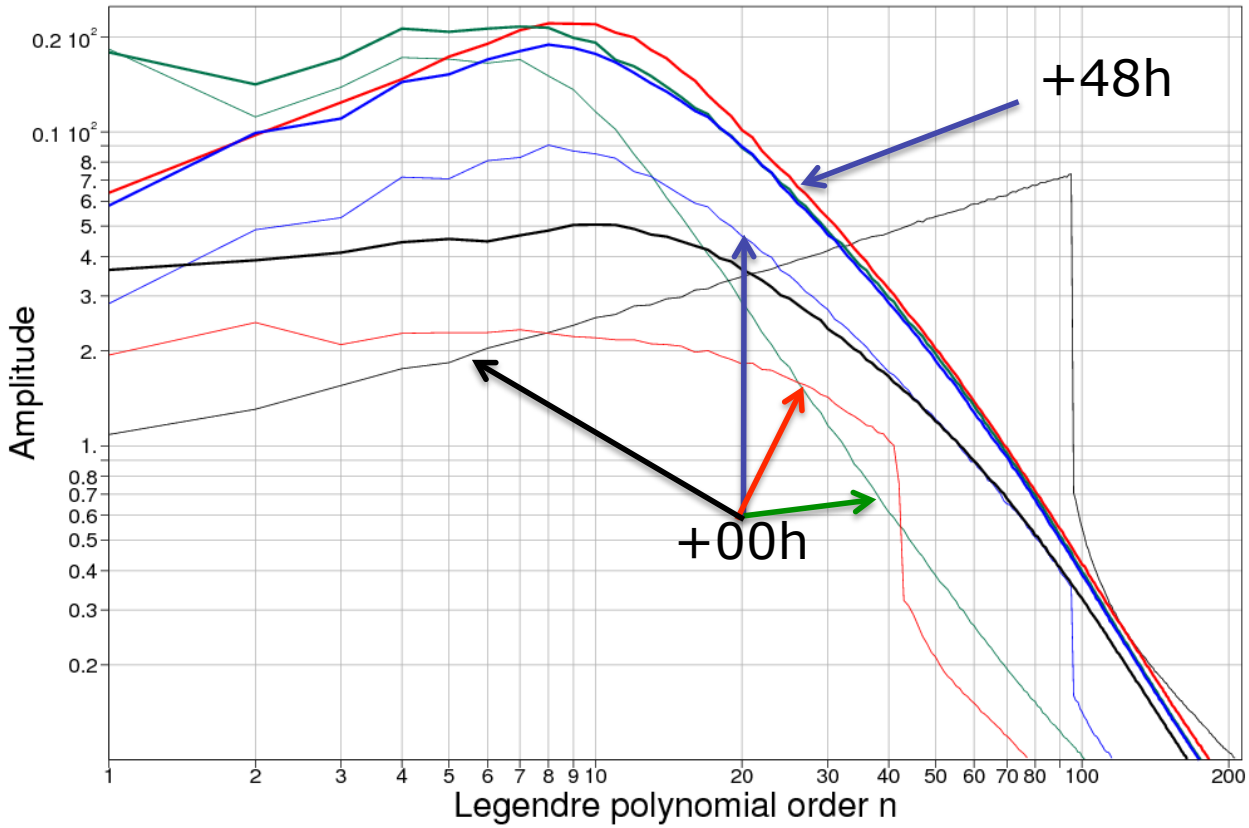
Perturbation amplitude spectra T700 +48



SV - Red
ET - Blue
RF – Green
RP - Black

Zonal mean perturbation energy (sqrt) Initially

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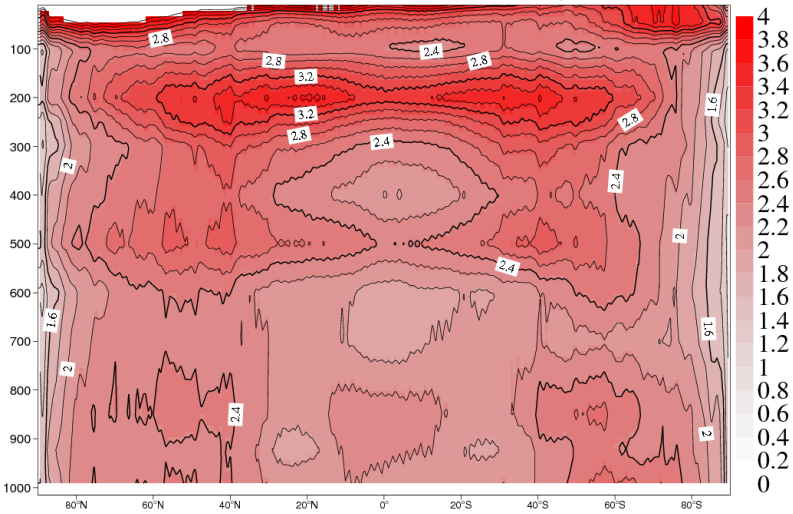
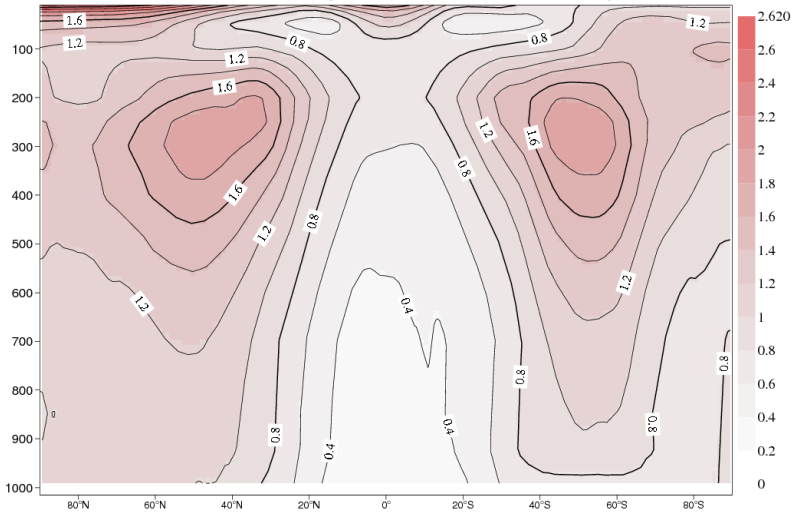
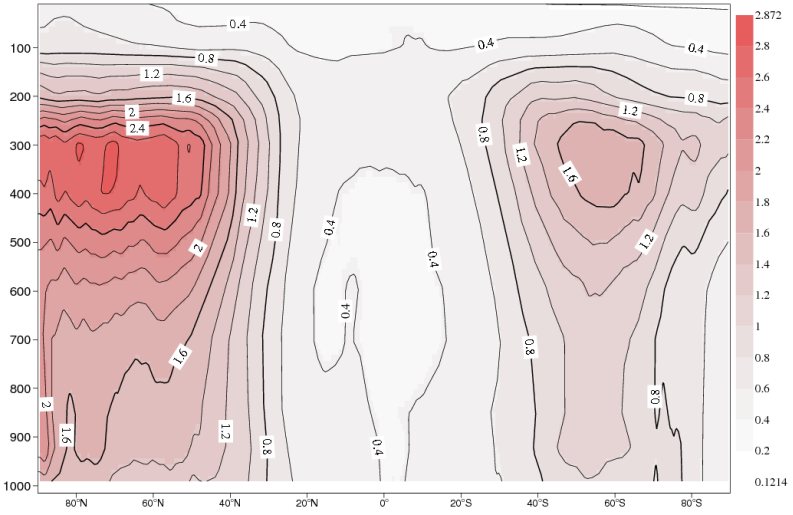
Random Perturbation Step 00



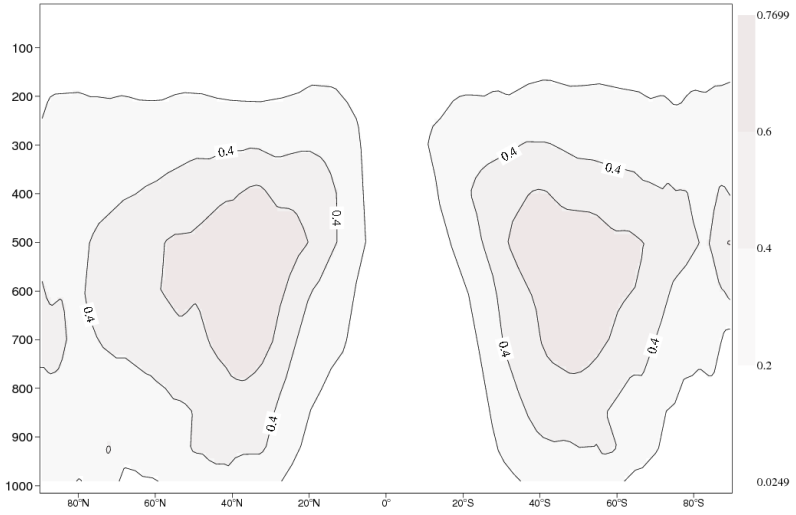
Random Field perturbations Step 00



Breeding Vectors Step 00



Singular Vectors Step 00



Zonal mean perturbation energy (sqrt) +48h

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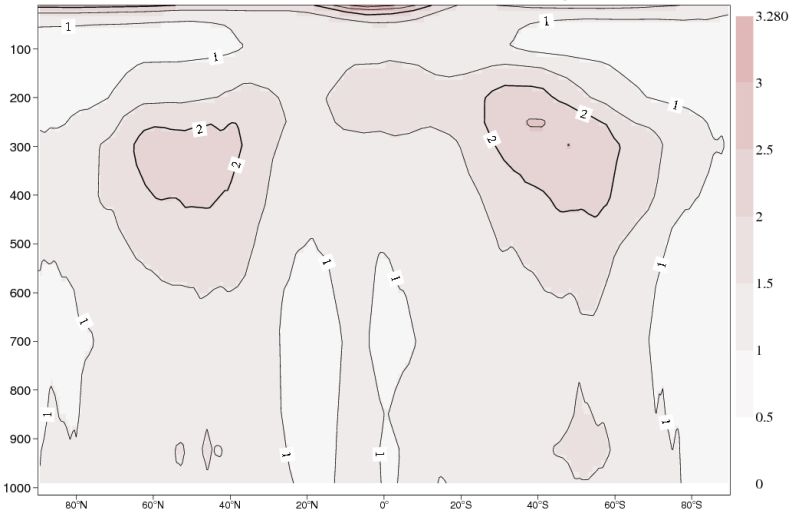
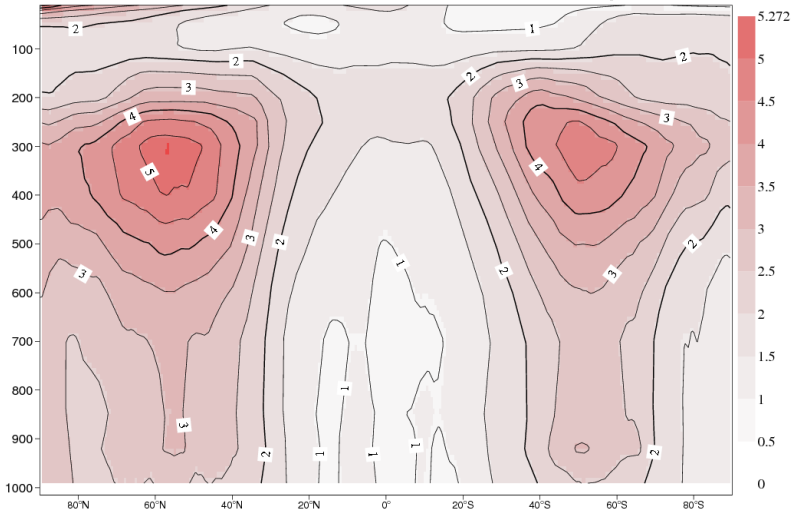
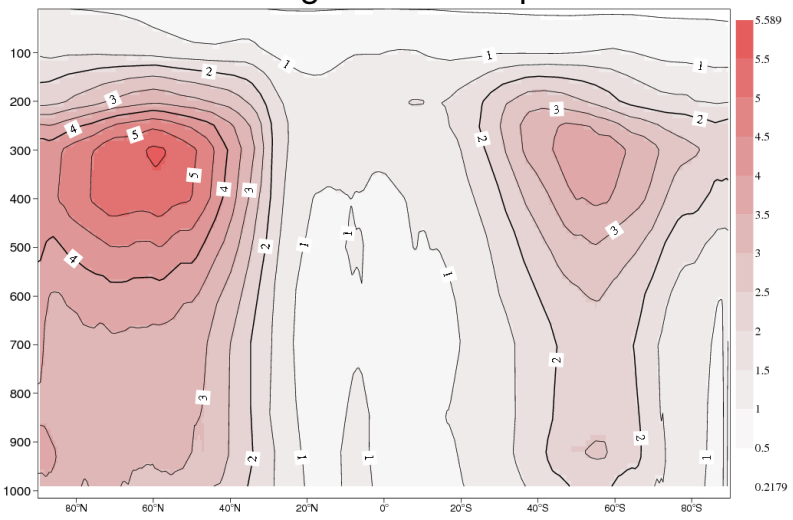
Random Perturbations Step 48



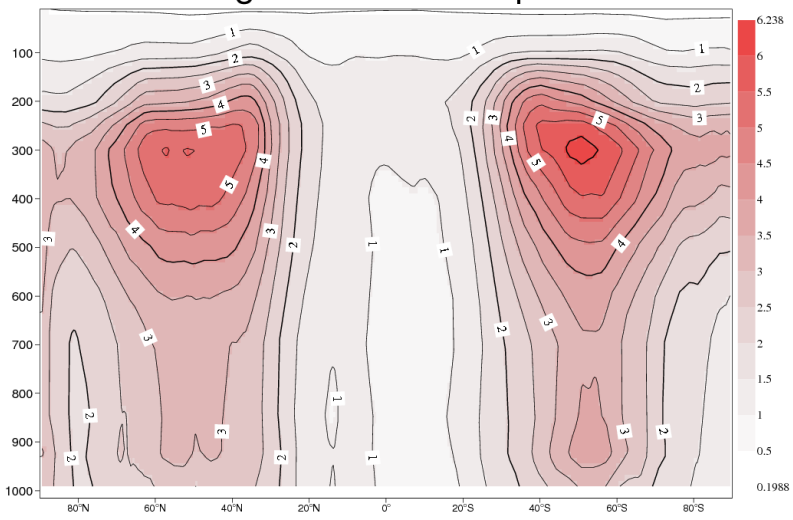
Random Field Perturbations Step 48



Breeding Vectors Step 48



Singular Vectors Step 48



Ensemble quality

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- Initial + evolved singular vectors
- Ensemble transform perturbations (regionally rescaled)
- 90 forecast cases
- 20 ensemble members
- ECMWF IFS-model T_L255L40

RMSE of Ensemble mean and Ensemble spread z500

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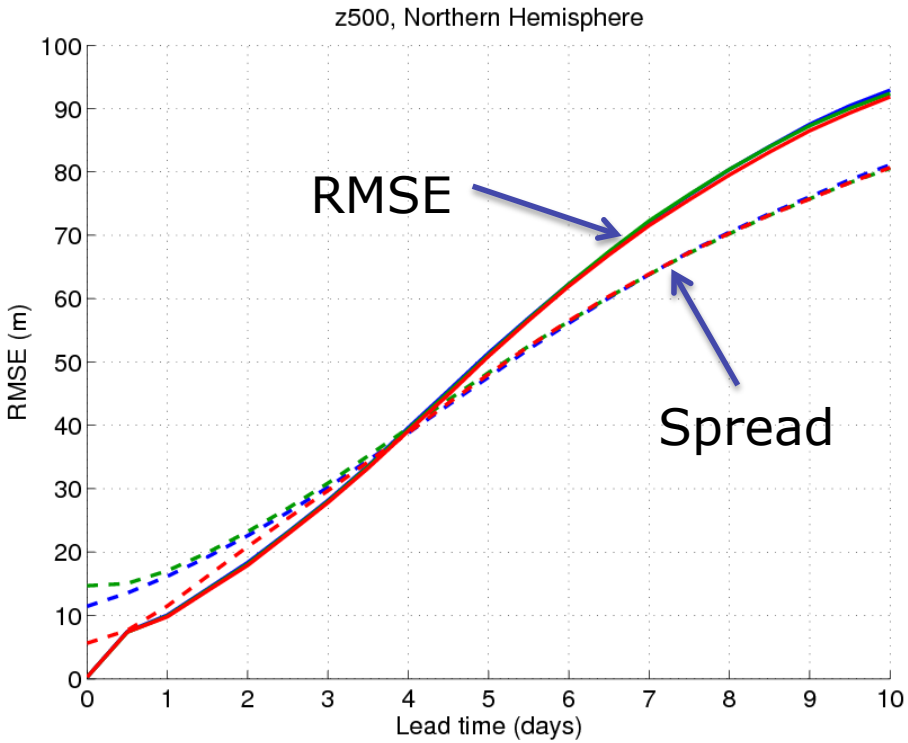
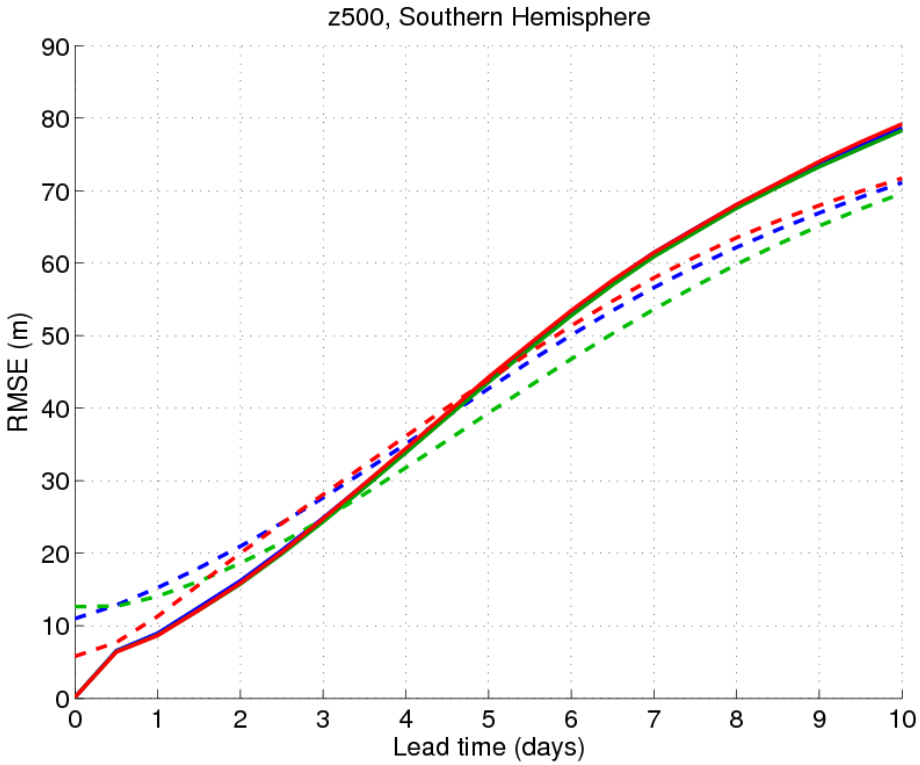
N.Hem

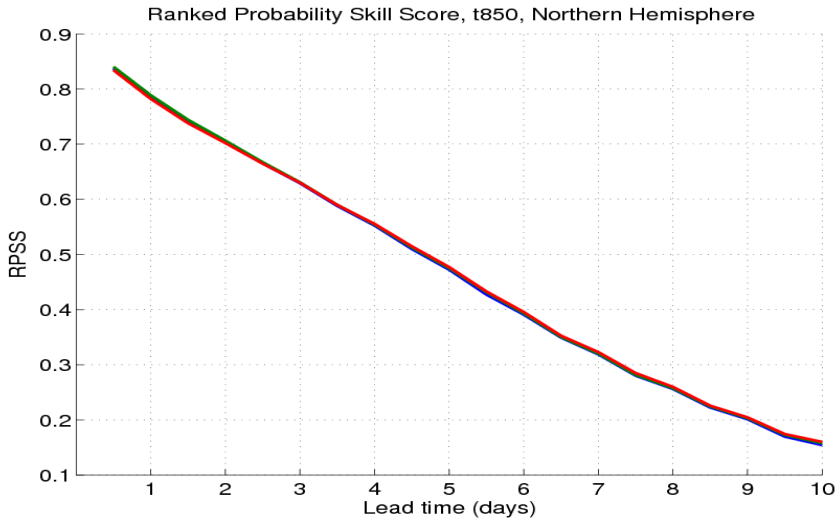


S.Hem



SV – Red ET – Blue RF - Green

Ranked Probability skill score t850



Different initial perturbations (N.Hem)

SV - Red
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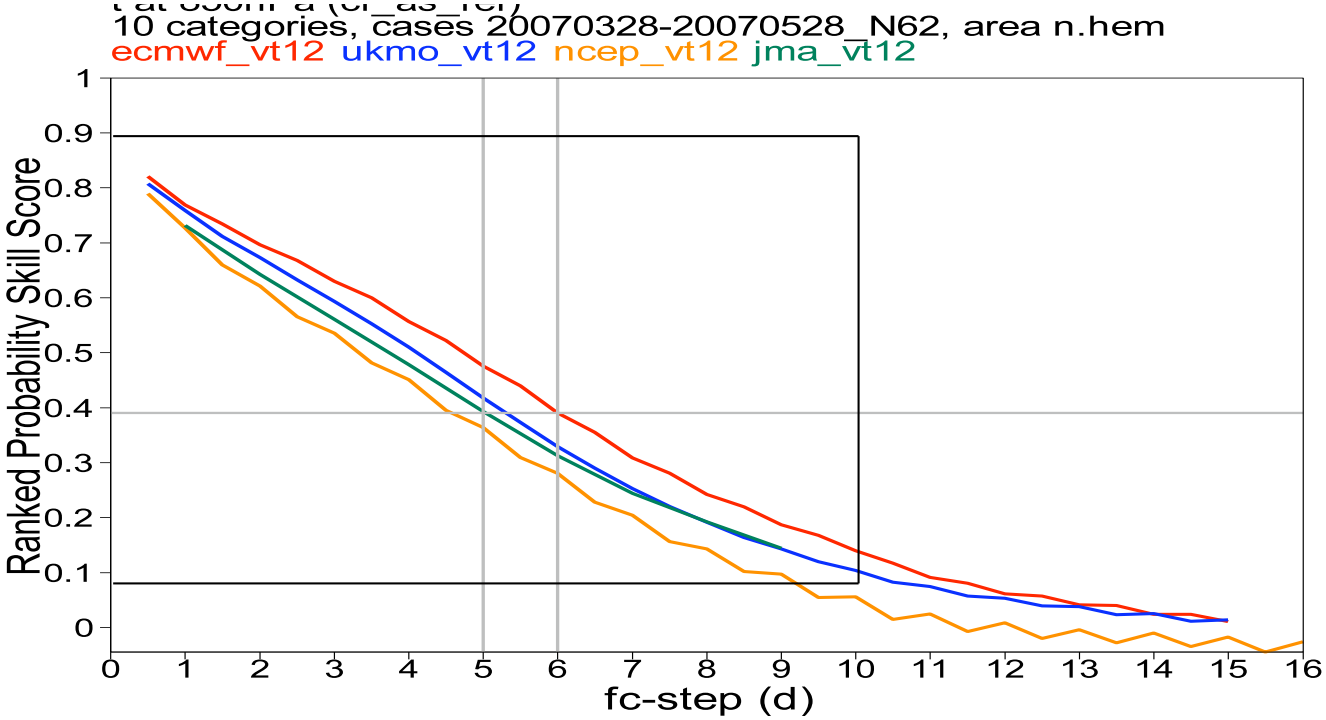
RF – Green
RP - Black



Different Centres (from Park et al.(2008), Courtesy R. Buizza)

Summary

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Singular Vectors

- Fast transient perturbation growth
- Strong connection to baroclinic zones
- Tropics?

Breeding Vectors

- Growth as Lyapunov exponent
- Mature structures
- Tropics?

Similar properties after the transient period

Discussion

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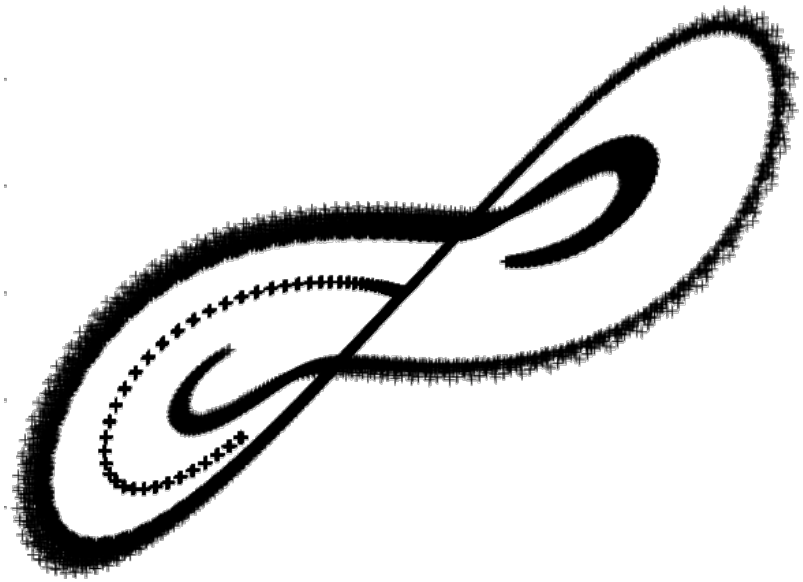
- Are the singular vectors too explosive?
- Are the breeding vectors too mature?
- Does it matter?



Papers:
Magnusson et al. (2009), Tellus
Magnusson et al. (2008), Monthly Weather Review
Magnusson et al. (2008), Nonlinear processes in geophysics

Perturbation Energy ratio

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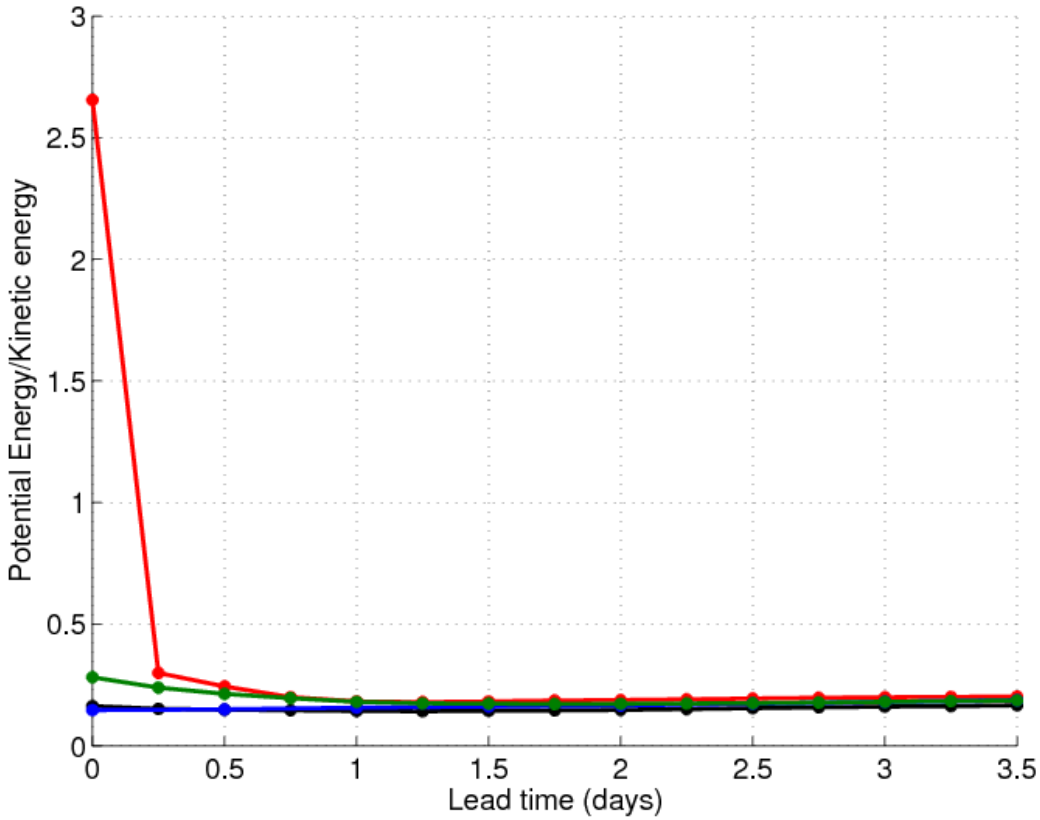

SV - Red
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