An Approach to Assess Observation Impact Based on Observation-minus-Forecast Residuals

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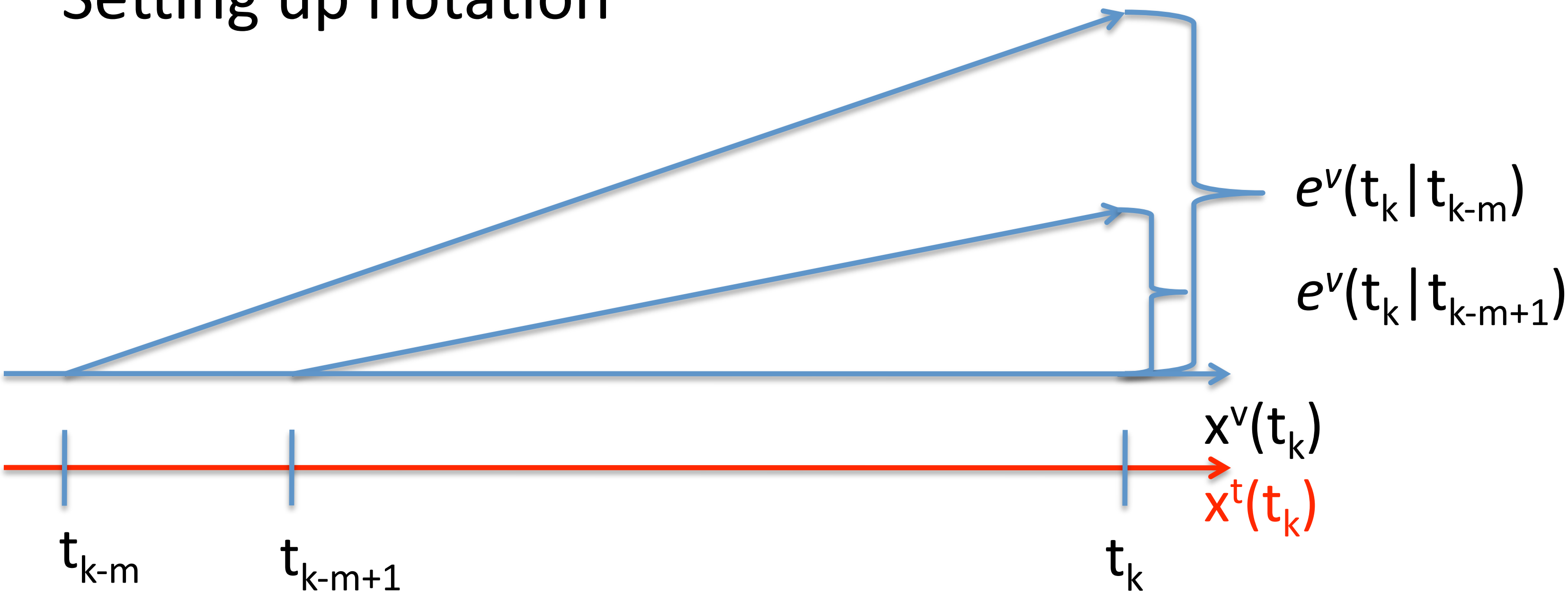
Two main points of this presentation

- To propose a more suitable choice of norm than the typical total energy-based norm.

- To suggest that an alternative simple methodology can be used to assess the impact of observations without need for the linear assumption and the practical complexities it introduces: need of adjoint, restricted applicability.

Setting up notation



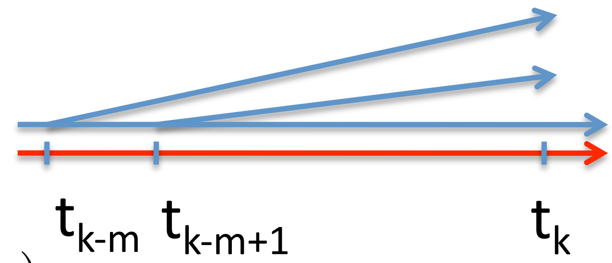
<u>Three error measures considered here:</u>

True state-space error: $$e_{k|\ell} \equiv (\mathbf{x}^f_{k|\ell} - \mathbf{x}^t_k)^T \mathbf{T}_k (\mathbf{x}^f_{k|\ell} - \mathbf{x}^t_k)$$

Perceived state-space: $$e^v_{k|\ell} \equiv (\mathbf{x}^f_{k|\ell} - \mathbf{x}^v_k)^T \mathbf{T}_k (\mathbf{x}^f_{k|\ell} - \mathbf{x}^v_k)$$

Observation-space residual: $$e^y_{k|\ell} \equiv \left[\mathbf{h}_k(\mathbf{x}^f_{k|\ell}) - \mathbf{y}^o_k\right]^T \mathbf{C}_k \left[\mathbf{h}_k(\mathbf{x}^f_{k|\ell}) - \mathbf{y}^o_k\right]$$

… Setting up notation …



Forecast model: $\mathbf{x}^f_{k|k-m+1} = \mathbf{m}_{k,k-m+1}(\mathbf{x}^a_{k-m+1|k-m+1})$

Suboptimal analysis update:

$$\mathbf{x}^a_{k-m+1|k-m+1} = \mathbf{x}^b_{k-m+1|k-m} + \tilde{\mathbf{K}}_{k-m+1|k-m}[\mathbf{y}^o_{k-m+1} - \mathbf{h}_{k-m+1}(\mathbf{x}^b_{k-m+1|k-m})]$$

OMB residual vector: $\mathbf{d}_{k-m+1|k-m} \equiv \mathbf{y}^o_{k-m+1} - \mathbf{h}_{k-m+1}(\mathbf{x}^b_{k-m+1|k-m})$

<u>State-space forecast error reduction</u>: $\delta e^v_k \equiv e^v_{k|k-m+1} - e^v_{k|k-m}$

<u>Observation-space forecast error reduction</u>: $\delta e^y_k \equiv e^y_{k|k-m+1} - e^y_{k|k-m}$

Remarks:

- The sub-optimality of the analysis update accommodates the weakly non-linear case
- Here, we'll be talking about the 1-day forecast error and corresponding error reduction

State-space (Adjoint) Approach

Question 1: How does the forecast error change with a change of initial condition?

Answer to Q1: Treat change of initial condition as infinitesimal and derive approximate formulae expressing the change in forecast error to various orders of accuracy. For example, a first-order expression involves:

$$\nabla_{\mathbf{x}^g} e^v_{k|\ell} \;=\; 2\mathbf{M}^T_{g;k,k-m+1}\mathbf{T}_k[\mathbf{x}^f_{k|\ell} - \mathbf{x}^v_k]$$

$$\mathbf{M}_{g;k,k-m+1} = \left.\frac{\partial \mathbf{m}_{k,k-m+1}(\mathbf{x})}{\partial \mathbf{x}}\right|_{\mathbf{x}=\mathbf{x}^g}$$

Question 2: How does the forecast error change when the initial condition changes as a consequence of assimilating observations?

Answer to Q2: Similarly to addressing Q1, treat change of initial condition as infinitesimal and derive approximate formulae expressing the change in forecast error to various orders of accuracy. For example, first and second order approximations give:

$$\delta e^{v,1}_k \;=\; \mathbf{d}^T_{k-m+1|k-m}\tilde{\mathbf{K}}^T_{k-m+1|k-m}\nabla_{\mathbf{x}^b} e^v_{k|k-m}\,,$$

$$\delta e^{v,2}_k \;=\; \frac{1}{2}\mathbf{d}^T_{k-m+1|k-m}\tilde{\mathbf{K}}^T_{k-m+1|k-m}\left[\nabla_{\mathbf{x}^b} e^v_{k|k-m} + \nabla_{\mathbf{x}^a} e^v_{k|k-m+1}\right]$$

Limitations to the state-space (adjoint) approach

- Practical definition of the forecast aspect requires verification state typically introducing what might be potentially undesirable correlations between residuals and forecasts

- Linearization assumptions constraint the technique to have limited applicability

- As used in practice, it infers statistical properties, but lacks suitable probabilistic framework

Observation-space Approach

Calculating the error reduction requires no approximation: $\delta e_k^y \equiv e_{k|k-m+1}^y - e_{k|k-m}^y$

Following Errico (2007) or Daescu and Todling (2009) one could write down approximations to the error reduction to various orders, but these are unnecessary.

Advantages of observation-space approach

- Verifying against observations avoids introduction of spurious effects in the forecast aspect
- No linearization and adjoints are need and therefore method is applicable to any length of forecast – no issues with multiple loops in the analysis minimization scheme
- Based on estimation theory and probabilistic approach
- Essentially cost-free and simple to implement

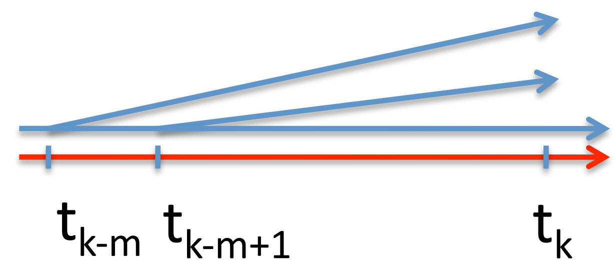
Insights on State- vs Observation-space Approaches

For the sake of argument, consider the linear case.

Define the forecast error covariance difference: $\mathbf{\Delta P}_k^f \equiv \mathbf{P}_{k|k-m+1}^f - \mathbf{P}_{k|k-m}^f$

Then:

$$
\begin{aligned}
< \delta e_k > &= \mathrm{Tr}\left\{\mathbf{T}_k \mathbf{\Delta P}_k^f\right\} \\
< \delta e_k^y > &= \mathrm{Tr}\left\{\mathbf{H}_k^T \mathbf{C}_k \mathbf{H}_k \mathbf{\Delta P}_k^f\right\}
\end{aligned}
$$



Remark: Probabilistic approach has very clear notion of improvement: $\mathbf{\Delta P}_k^f < \mathbf{0}$

Useful definitions for what follows:

Observation-minus-forecast residual covariance matrix:

$$
\mathbf{\Gamma}_{k|k-m} = \mathbf{H}_k \mathbf{P}_{k|k-m}^f \mathbf{H}_k^T + \mathbf{R}_k
$$

Difference between a general, *suboptimal* gain, and the Kalman gain matrices:

$$
\mathbf{\Delta K}_k \equiv \tilde{\mathbf{K}}_k - \mathbf{K}_k
$$

Insights on State- vs Observation-space Approaches

One can derive the following basic results:

1. For optimal systems, the *expected* forecast error reduction always corresponds to positive impact – assimilation of data always leads to improvement in the expected mean sense.

$$< \delta e_k > \overset{opt}{\leq} 0$$

$$< \delta e_k^y > \overset{opt}{\leq} 0$$

2. For optimal systems, and a suitable choice of weighting matrix **T**$_k$, the state-space *expected* forecast error reduction produces the same estimate as that obtained in observation-space.

$$< \delta e_k(\mathbf{T}_k = \mathbf{H}_k^T \mathbf{C}_k \mathbf{H}_k) > \overset{opt}{=} < \delta e_k^y >$$

Since $rank(\mathbf{T}_k) \geq rank(\mathbf{C}_k)$ there is only so much the measure in observation-space can capture when compared with that in state-space, however, the remaining part is not accessible to us.

Insights on State- vs Observation-space Approaches

3. In general, for suboptimal systems, verifying against a state other than the truth introduces a correlation (covariance) between the observation-minus-background residual and the error in the verification, $\boldsymbol{\epsilon}_k^v \equiv \mathbf{x}_k^v - \mathbf{x}_k^t$

$$< \delta e_k^v > \;\; = \;\; < \delta e_k > -2\mathrm{Tr}\left[\tilde{\mathbf{K}}_{k-m+1}^T \mathbf{M}_{k,k-m+1}^T \mathbf{T}_k < \boldsymbol{\epsilon}_k^v \mathbf{d}_{k-m+1|k-m}^T > \right]$$

4. In general, for suboptimal systems, if the verification is chosen to be the underlying analysis all intermediate residual correlations (covariances) participate

$$\begin{aligned} < \delta e_k^{v=a} > \;\; &= \;\; < \delta e_k > \\ & -2\mathrm{Tr}\left[\tilde{\mathbf{K}}_{k-m+1}^T \mathbf{M}_{k,k-m+1}^T \mathbf{T}_k \left(\mathbf{M}_{k,k-m+1} \Delta\mathbf{K}_{k-m+1} \boldsymbol{\Gamma}_{k-m+1} \right.\right. \\ & \left.\left. + \sum_{j=0}^{m-2} \mathbf{M}_{k,k-j} \tilde{\mathbf{K}}_{k-j} < \mathbf{d}_{k-j|k-j-1} \mathbf{d}_{k-m+1|k-m}^T > \right)\right] \end{aligned}$$

5. <u>Therefore, only in the optimal case, use of the verification is equivalent to use of the unknown true state to obtain the expected error change of interest</u>.

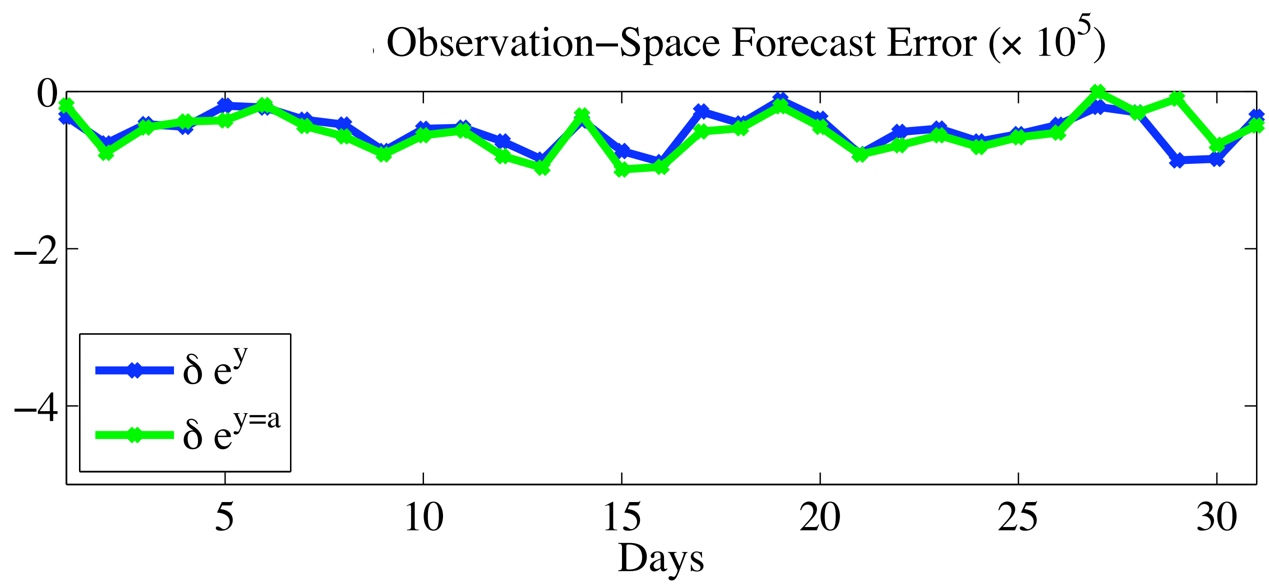
Experimental Setup

- Test-bed: GEOS-5 DAS 2x2.5x72
- Observation impact on all 24-hr forecasts from 00UTC for August 2007
- Broad LPO, excluding only very top layers of model
- What follows:
  1. Quick test for the role of the verification
  2. Compares the state-space (adjoint) approach for three different norms
  3. Compares results from observation-space approach to what's obtained in (2)

The Role of the Verification

The role of the verification can precisely tested in observation-space. Similarly, to the result obtained in state-space, when the verification is chosen to be the analysis, now projected onto observation space, the following holds:

$$< \delta e_k^{y=a} > \; = \; < \delta e_k^{y} > -2\mathrm{Tr}\left[\tilde{\mathbf{K}}^T_{k-m+1}\mathbf{M}^T_{k,k-m+1}\mathbf{H}_k^T\mathbf{C}_k\mathbf{H}_k < \boldsymbol{\epsilon}_k^{v=a}\mathbf{d}^T_{k-m+1|k-m} > \right]$$

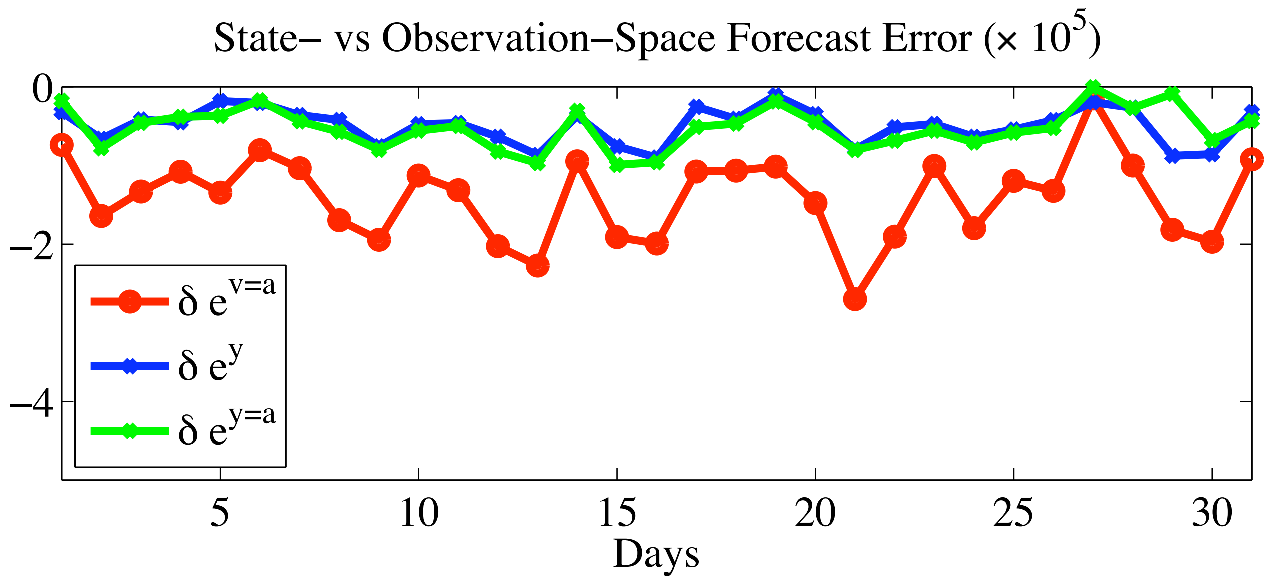


The result with GEOS-5 DAS indicates that <u>in the light of this global measure</u>, the system is nearly optimal, and using the analysis as a proxy for the observations is reasonable most of the time. Indeed, this provides a test of optimality.

State-space vs Observation-space Error Reduction

When the state-space measure is designed to capture what the observation-space measure captures it reveals the correlations between the observation errors and the OMB-residuals.

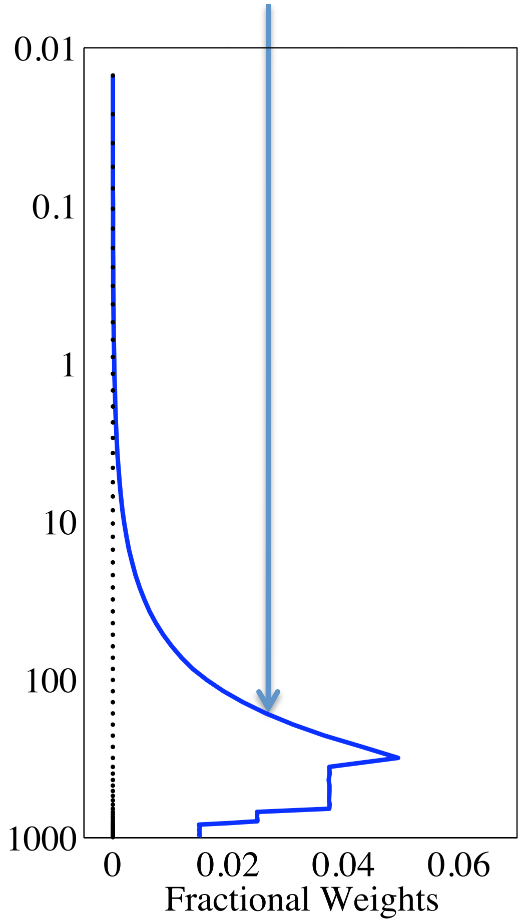
$$
\begin{aligned}
< \delta e_k^y > \;&\approx\; < \delta e_k^{y,1} > \\
&= \; < \delta e_k^{v,1} > -2\mathrm{Tr}\left[\tilde{\mathbf{K}}_{k-m+1}^T \mathbf{M}_{k,k-m+1}^T \mathbf{H}_k^T \mathbf{C}_k \mathbf{H}_k < \boldsymbol{\epsilon}_k^o \mathbf{d}_{k-m+1|k-m}^T >\right]
\end{aligned}
$$

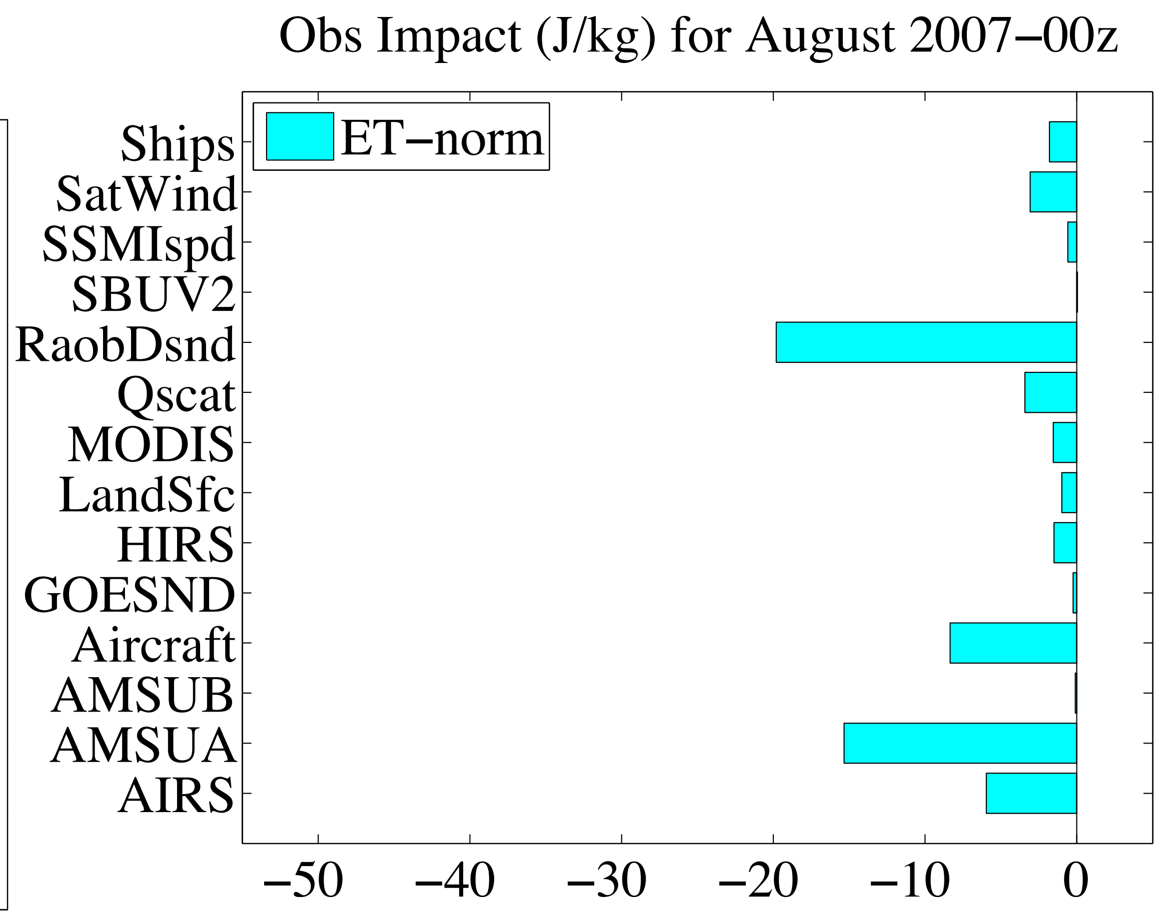


Observation Impacts and Choice of Norm

$$e_t \equiv \mathbf{x}_1^T \mathbf{T}_t \mathbf{x}_2 = \frac{1}{2} \sum_{i,j,k} \Delta H_{i,j} \underline{\Delta \sigma_{i,j,k}} \left[u_1' u_2' + v_1' v_2' + \frac{c_p}{T_r} T_1' T_2' + \frac{R T_r}{p_r^2} p_{s1}' p_{s2}' \right]_{i,j,k}$$

$$\Delta \sigma_{i,j,k} = \frac{\Delta p_{i,j,k}}{p_{s\,,i,j} - p_t}$$

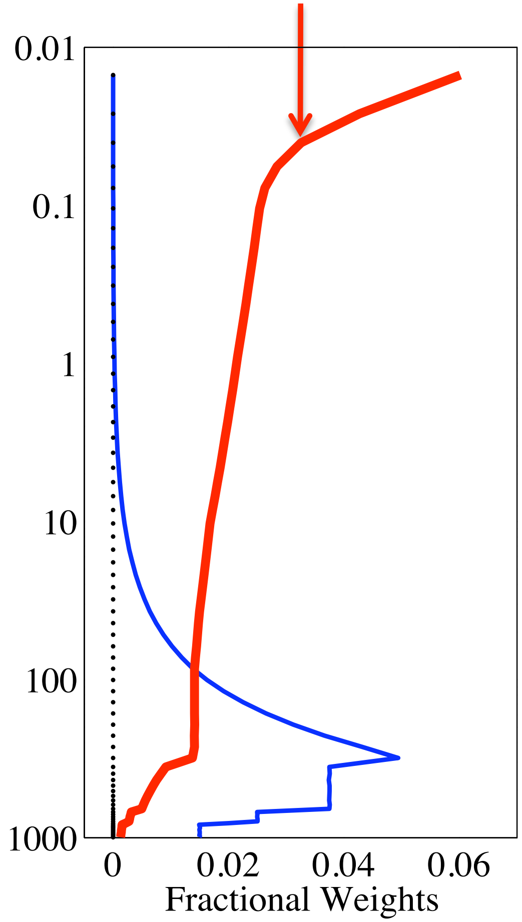


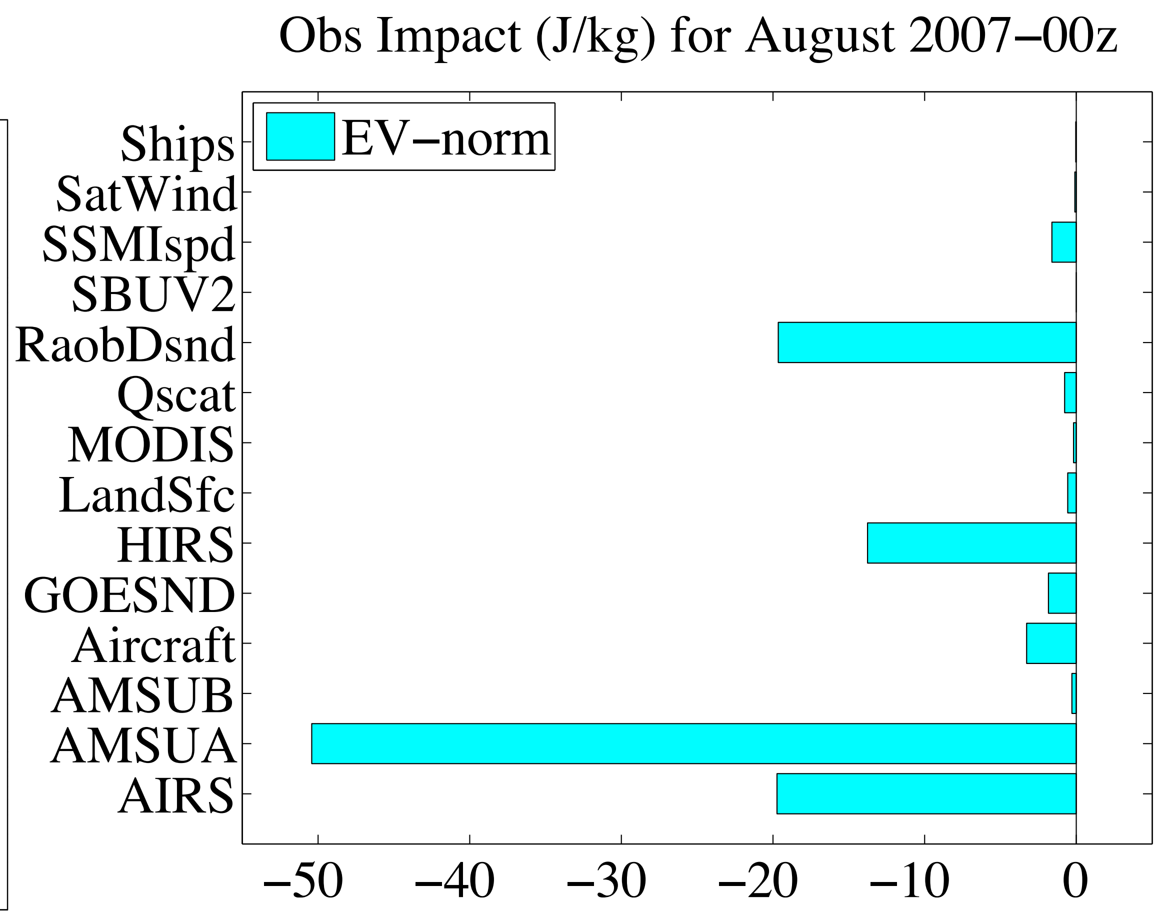


Observation Impacts and Choice of Norm

$$e_s \equiv \mathbf{x}_1^T \mathbf{T}_s \mathbf{x}_2 = \frac{1}{2} \sum_{i,j,k} \Delta H_{i,j} \underline{\Delta z_{i,j,k}} \left[u_1' u_2' + v_1' v_2' + \frac{c_p}{T_r} T_1' T_2' + \frac{R T_r}{p_r^2} p_{s1}' p_{s2}' \right]_{i,j,k}$$

$$\Delta z_{i,j,k} = \frac{\Delta \ln p_{i,j,k}}{\ln p_{s,i,j} - \ln p_t}$$



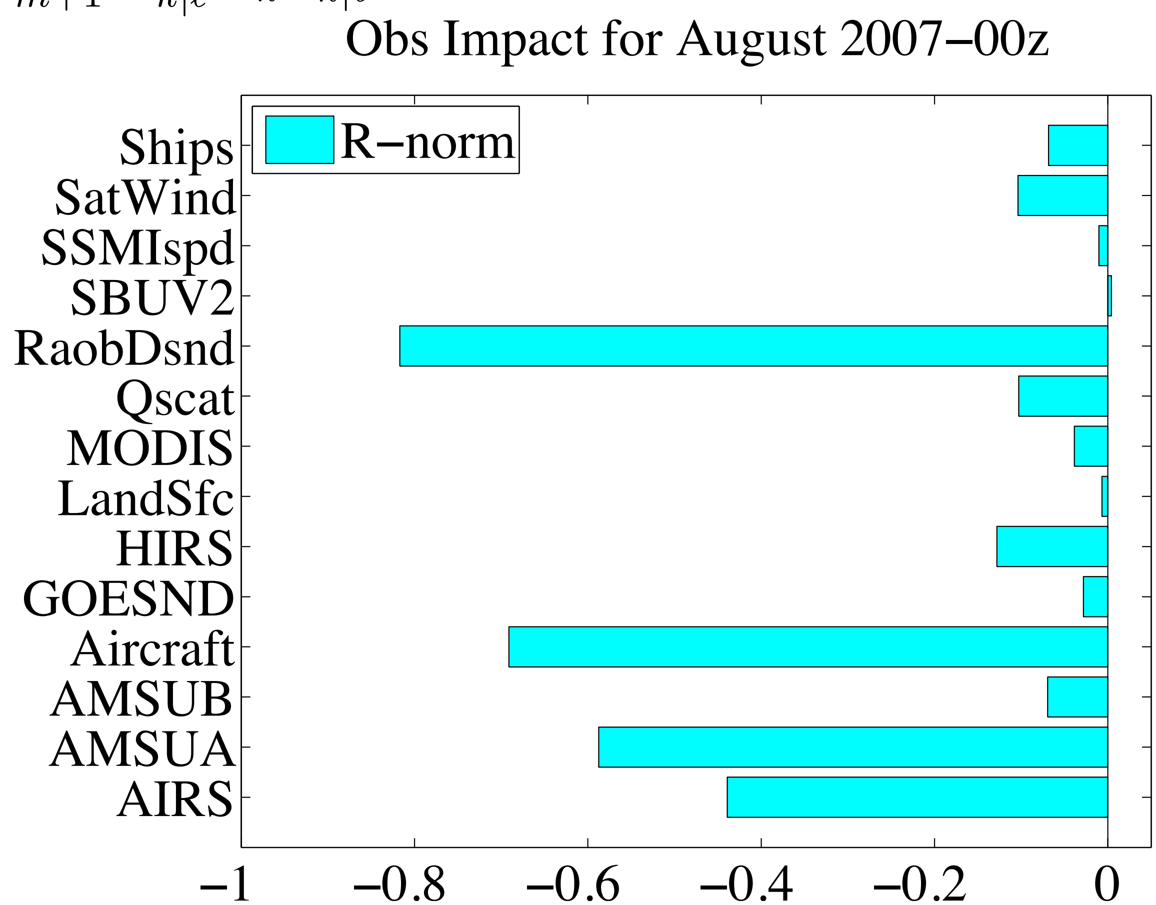


Observation Impacts and Choice of Norm

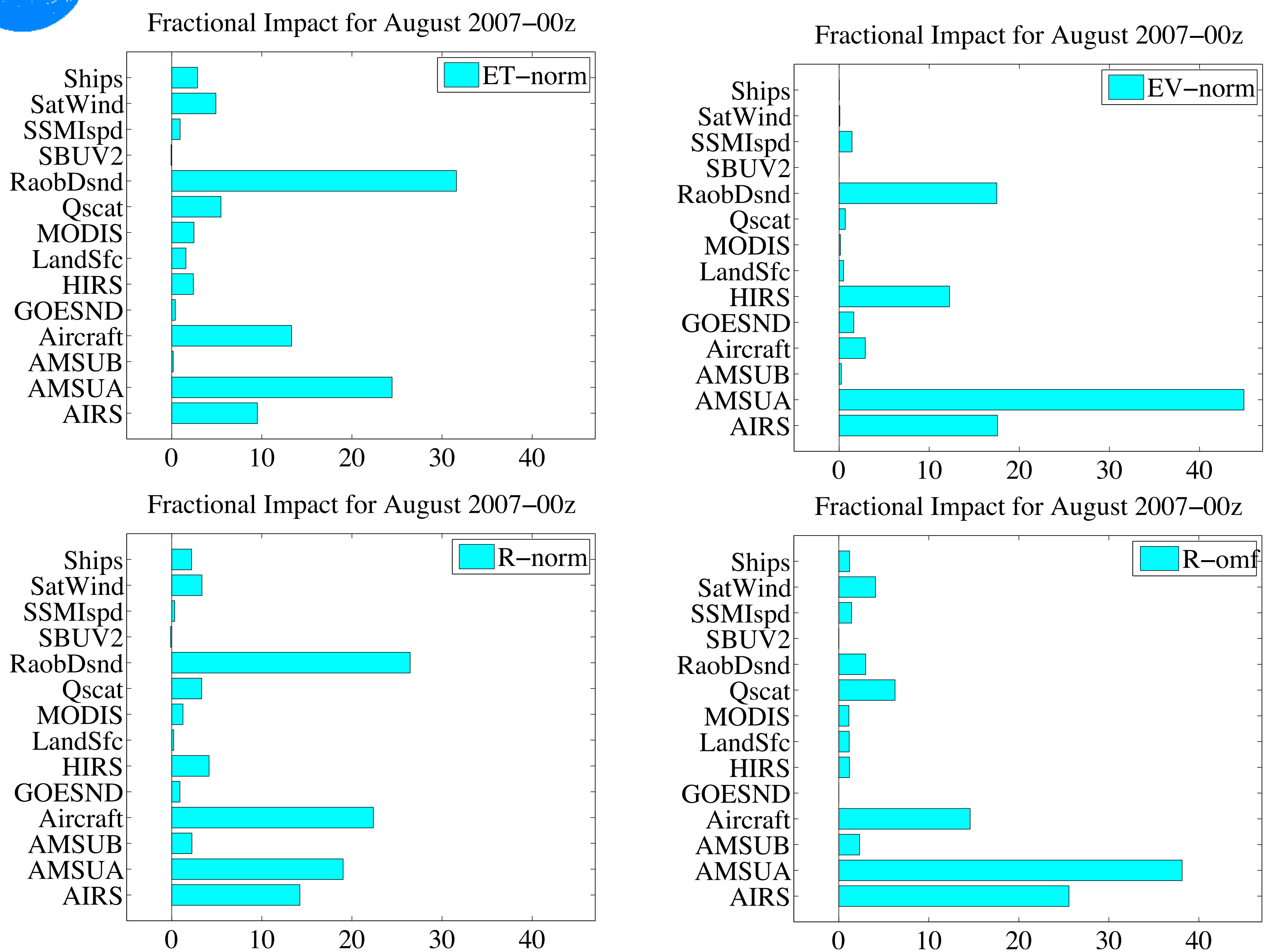
$$\delta e_k^y \approx \delta e_k^{y,2} \;=\; -\frac{1}{2}\mathbf{d}_{k-m+1|k-m}^T \tilde{\mathbf{K}}_{k-m+1|k-m}^T \left[\nabla_{\mathbf{x}^b} e_{k|k-m}^y + \nabla_{\mathbf{x}^a} e_{k|k-m+1}^y\right]$$

$$\nabla_{\mathbf{x}^g} e_{k|\ell}^y \;=\; -2\mathbf{M}_{g;k,k-m+1}^T \mathbf{H}_{k|\ell}^T \mathbf{C}_k \mathbf{d}_{k|\ell}$$

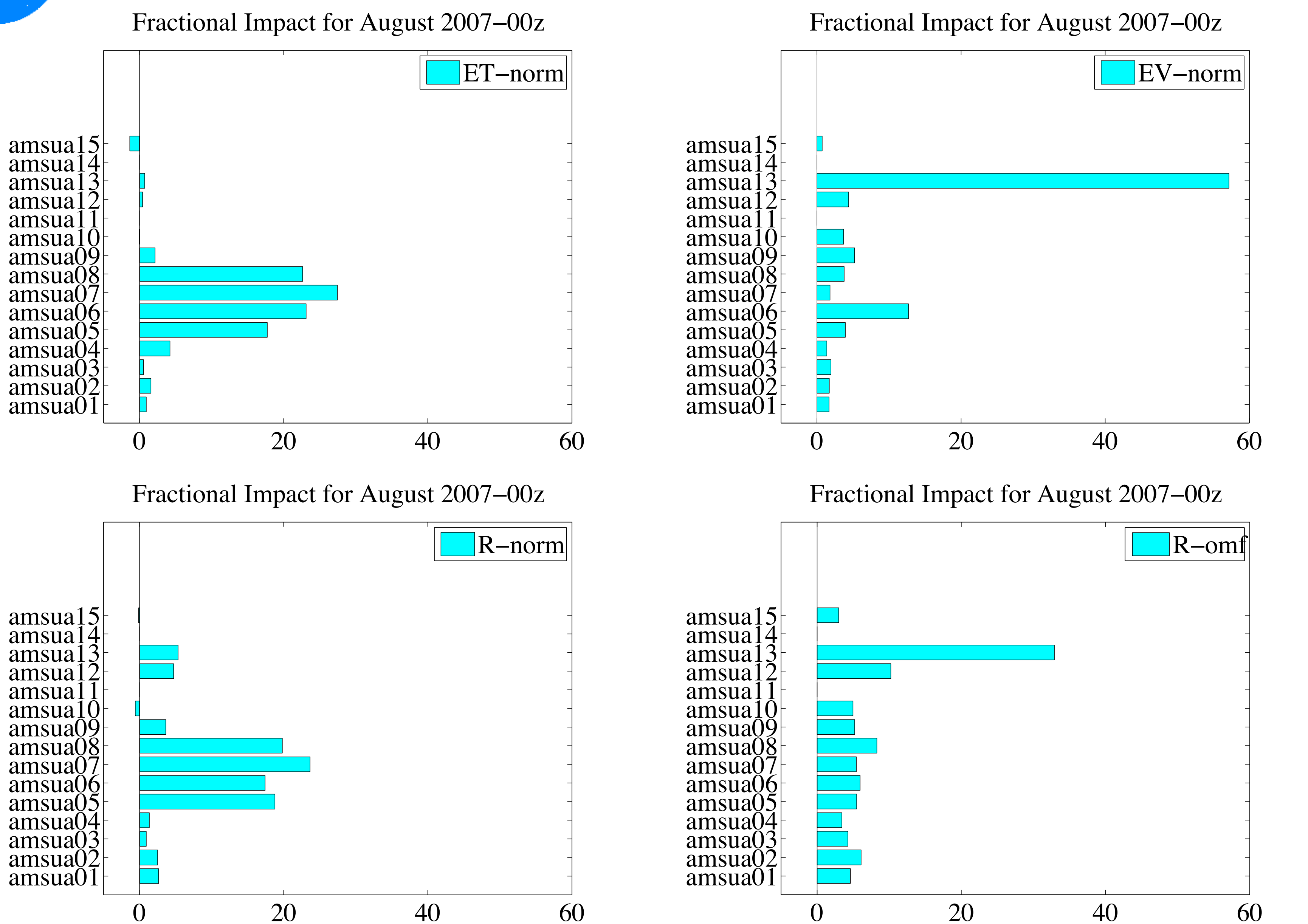
$$\mathbf{C}_k = \mathbf{R}_k^{-1}$$



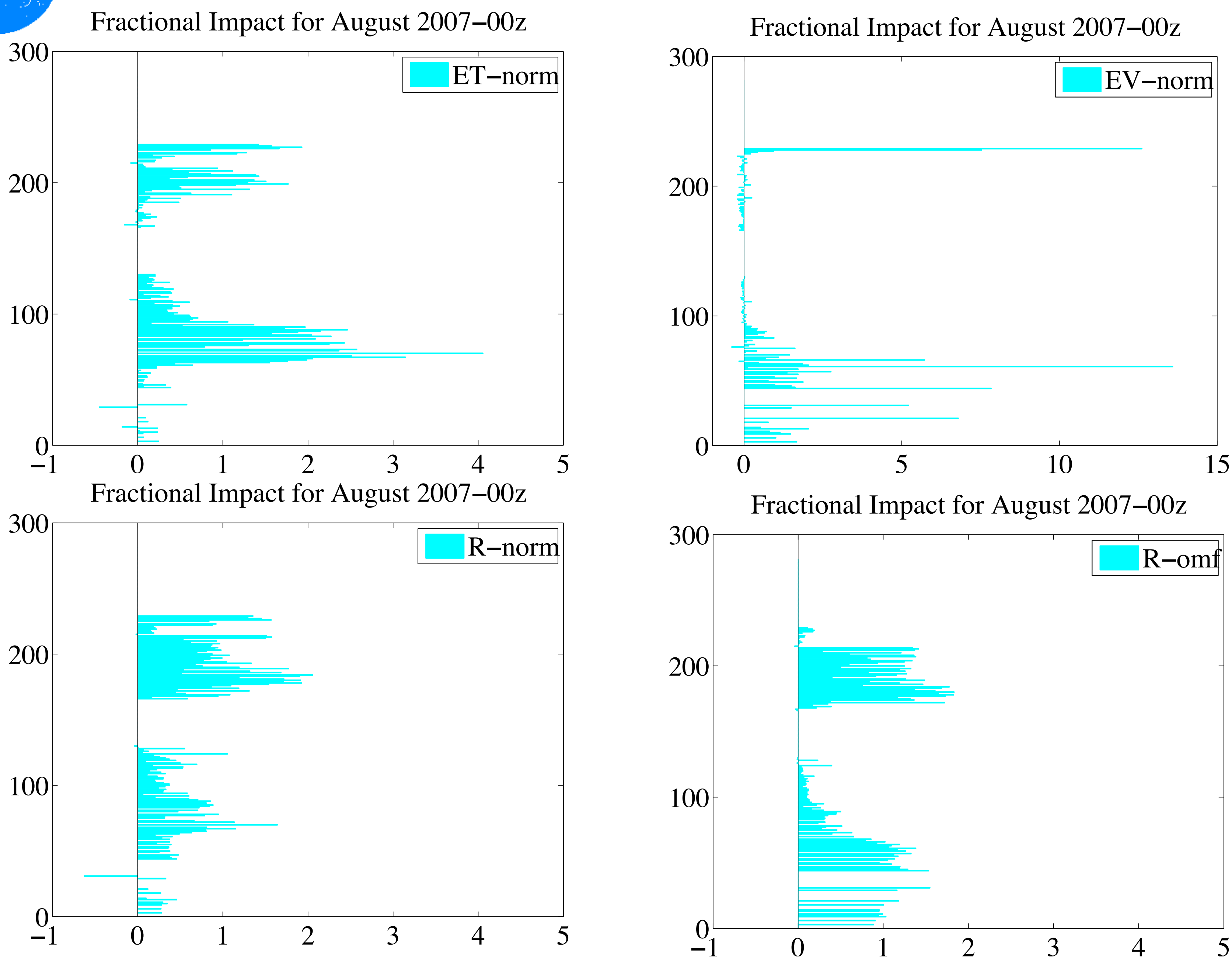
Overall Fractional Impacts for various error measures



Fractional Impacts for AMSU-A on NOAA-15 and 16



Fractional Impacts for AIRS on AQUA



Limitations of observation-space approach

- Observation-space measures only capture a part of the forecast error – that part projecting on the space of observations – unfortunately, this is *only* part accessible to us.
- In practice, since observations are bias-corrected, there is still a correlation in the forecast aspect between the forecast and the verification (i.e., bias-corrected observations in this case).
- The observing system is assumed to be relatively homogenous in time.

Conclusions

- A fair assessment of the observing system impact on the forecast requires careful choice of a forecast error measure.
- Impacts derived from the sequence of observation-minus-forecast residuals provide similar information to that obtained with adjoint-based techniques with considerably less restrictions and complexity.

"I like the dreams of the future better than the history of the past."

- Thomas Jefferson