Development of 4DEnVar at Météo-France

G. Desroziers, E. Arbogast, L. Berre

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Outline

1. Introduction
2. Operational configuration
3. 4DEnVar development
4. Localization with the time dimension
5. Conclusion

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Introduction

- 4D-Var

✓ Possible improved representation of **B** with an ensemble of 4D-Vars (Météo-France, ECMWF):
"flow-dependent" error variances and
correlations, with a wavelet **B** (Fisher, 2003; Varella et al, 2011).

✓ However, difficult development and maintenance of TL/AD.
✓ Low scalability of TL/AD at low resolution.

- 4D-Var based on a 4D ensemble: 4DEnVar

✓ Avoids TL/AD forecast models.
✓ Similar to 4D-Var (**H**, global analysis, additional terms, outer loops, …).
✓ Localization in model space (versus observation space for EnKF).
✓ Lower cost than 4D-Var and parallelization possibilities.

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Current ARPEGE global assimilation configuration

- ”Deterministic” 4D-Var

✓ 6 hour time window.

✓ 2 outer loops

T1198 C2.2 (7.5 km min) L105 / T149 (~135 km), T399 (~ 50 km).

✓ Jc-DFI, VarBC.

✓ $\mathbf{B}^{1/2} = \mathbf{K}^b \Sigma^b \mathbf{C}^{1/2}$, wavelet **C**, $\mathbf{K}^b$ = spectral + non-linear balances.

- Ensemble assimilation

✓ 25 perturbed 4D-Vars.

✓ 1 outer loop T479 C1.0 (40 km) / T149 C1.0.

✓ Multiplicative inflation of 3h forecast perturbations.

✓ Gives

➢ filtered Σ^b from the last 25 perturbations, updated every 6 h,

➢ wavelet **C** from the last 6 x 25 perturbations (last 30 h), updated every 6 h.

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4DEnVar formulation

- Minimization of

$$J(\underline{\delta\mathbf{x}}) = \underline{\delta\mathbf{x}}^{T}\, \underline{\mathbf{B}}^{-1}\, \underline{d\mathbf{x}} + (\underline{\mathbf{d}}\text{-}\underline{\mathbf{H}}\ \underline{\delta\mathbf{x}})^{T}\, \underline{\mathbf{R}}^{-1}\, (\underline{\mathbf{d}}\text{-}\underline{\mathbf{H}}\ \underline{\delta\mathbf{x}}), \text{ with } \underline{\mathbf{B}} = \underline{\mathbf{X}}^{b'}\, \underline{\mathbf{X}}^{b'T},$$

$\underline{\mathbf{X}}^{b'} = (\underline{\mathbf{x}}^{b'}_{1}, \ldots, \underline{\mathbf{x}}^{b'}_{Ne})$, and $\underline{\mathbf{x}}^{b'}_{ne} = \underline{\mathbf{x}}^{b}_{ne} - <\underline{\mathbf{x}}^{b}> / (N^{e}-1)^{1/2}$, ne =1, N^{e}.

$\underline{\mathbf{x}}^{b'}$ with dimension K+1 (times) x M (3D variables) x N (3D dimension)

(Liu et al, 2008, 2009; Buehner et al, 2010; Lorenc, 2012).

- Formulation developed at Météo-France

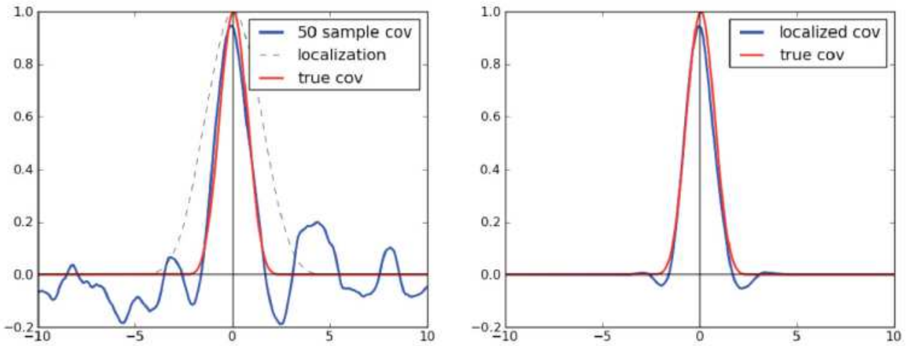
✓ $\underline{\delta\mathbf{x}}$ as a control variable (no α or χ variables) (Desroziers et al, 2014).

✓ DPCG minimizer (Derber and Rosati, 1989) with $\underline{\mathbf{h}} = \underline{\mathbf{B}}\ \underline{\mathbf{g}}$ operations.

✓ Also possible in observation space with $\underline{\mathbf{h}}^{y} = \underline{\mathbf{H}}\ \underline{\mathbf{B}}\ \underline{\mathbf{H}}^{T}\ \underline{\mathbf{g}}^{y}$ operations.

Localization of ensemble covariances

- Need for localization



(Whitaker, 2011)

- Simplication of the localization

Same **L** for all 3D variables and times: $\underline{\mathbf{B}} = \underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T} \circ \underline{\mathbf{L}}$, with

$$\underline{\mathbf{L}} = \begin{pmatrix} \mathbf{L} & \mathbf{L} & & \mathbf{L} \\ \mathbf{L} & & & \\ & & & \\ \mathbf{L} & & & \mathbf{L} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{I} \\ \\ \mathbf{I} \end{pmatrix} \mathbf{L} \, (\mathbf{I} \;\; \mathbf{I} \qquad \mathbf{I}) = \underline{\mathbf{1}} \, \mathbf{L} \, \underline{\mathbf{1}}^T$$

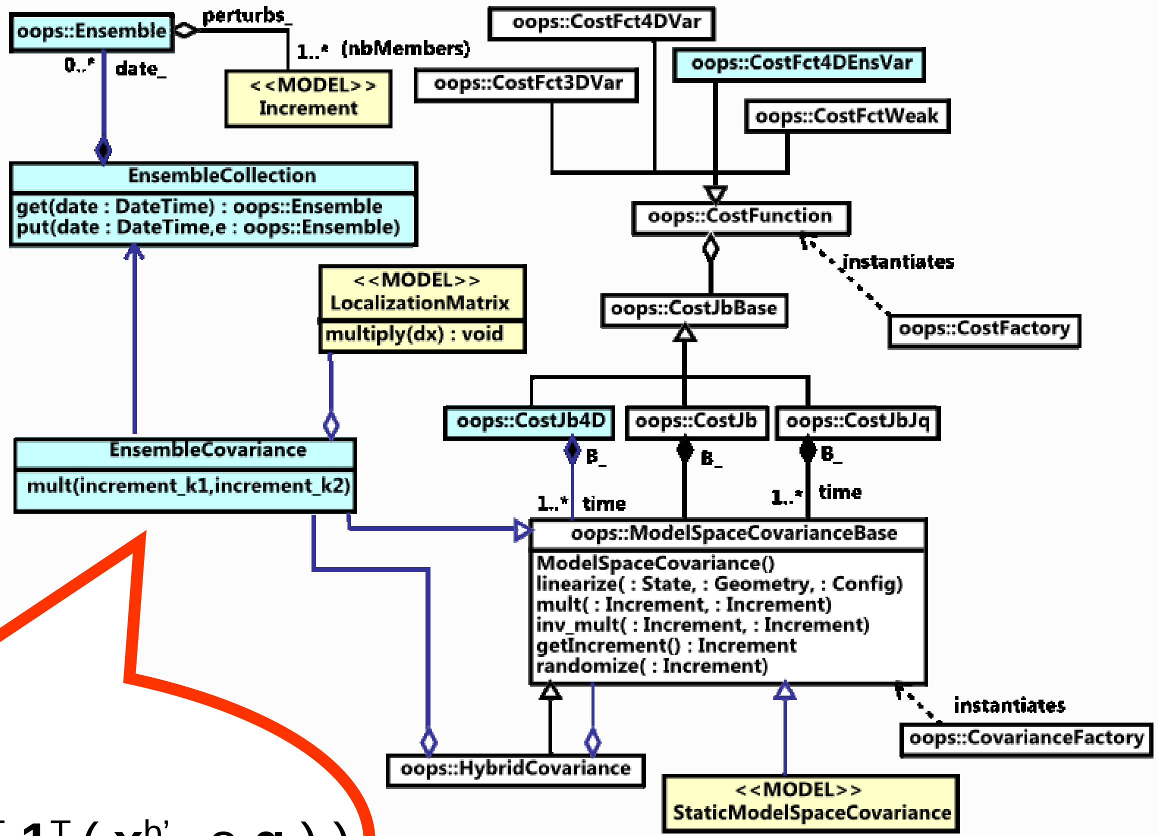
, and **I** is the NxN identity matrix.

Matrix $\underline{\mathbf{1}}$ contains K+1 (times) x M (var.) blocks **I** and **L** is a NxN correl. matrix.

- Applicat. of $\underline{\mathbf{B}}$: $\underline{\mathbf{h}} = \underline{\mathbf{B}} \, \underline{\mathbf{g}} = (\underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T} \circ \underline{\mathbf{L}}) \, \underline{\mathbf{g}} = \Sigma_{ne} \, \underline{\mathbf{x}}^{b'}_{ne} \circ (\underline{\mathbf{1}} \, \mathbf{S}^{-1} \, \mathbf{L} \, \mathbf{S}^{-T} \, \underline{\mathbf{1}}^T (\underline{\mathbf{x}}^{b'}_{ne} \circ \underline{\mathbf{g}})).$

4DEnVar under OOPS
(Object Oriented Prediction System, ECMWF/Météo-France)

Jb classes



$$\Sigma_{ne}\, \underline{\mathbf{x}}^{b'}_{ne} \circ \left(\underline{\mathbf{1}}\, \mathbf{S}^{-1}\, \mathbf{L}\, \mathbf{S}^{-T}\, \underline{\mathbf{1}}^{T} \left(\underline{\mathbf{x}}^{b'}_{ne} \circ \underline{\mathbf{g}} \right) \right)$$

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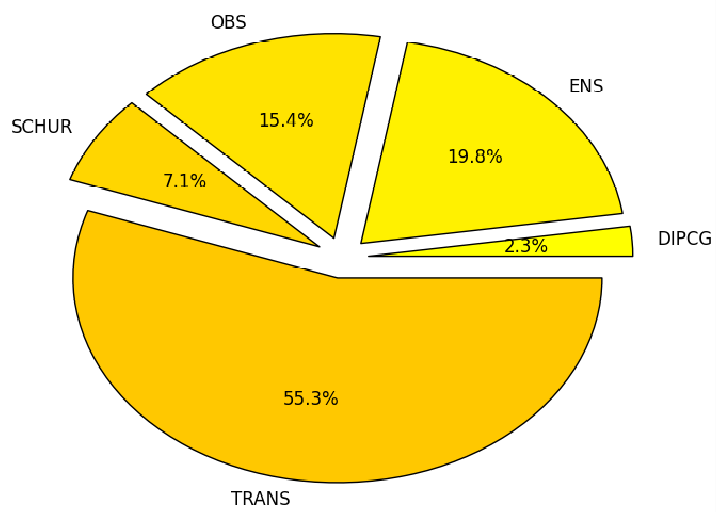
Optimization of the computations

- Geographical parallelization with **MPI**.
- Gridpoint and spectral parallelization with **OpenMP**.
- $\underline{\mathbf{x}}^{b'}$ reading parallelization with **a pool of C++ threads**.
- Computation cost with $N^e = 200$, T399, 40 it,

  75 nodes, 150 MPI tasks, 12 OpenMP threads by task: **9'**

  with conventional observations only.



Optimization of the localization

- Transform ψ, χ and Ps to have better agreement between all variables

ψ —–> $\Delta^{1/2}\psi$

χ —–> $\Delta^{1/2}\chi$

T

q

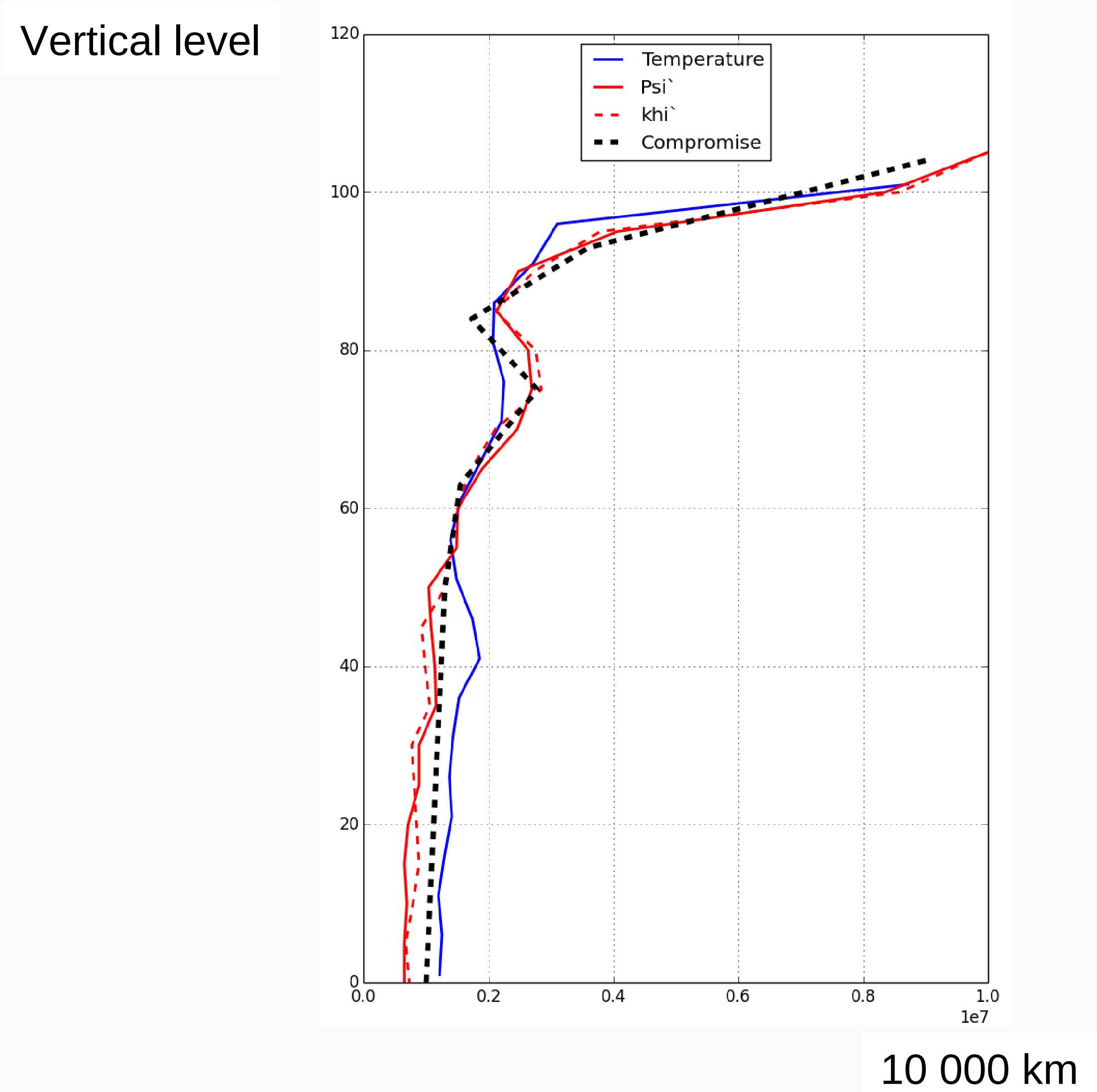
Ps —–> $\Delta^{1/2}$ Ps

- Localized matrix with transformed variables

$$\underline{\mathbf{B}} = \underline{\mathbf{U}}^{-1} \left(\left(\underline{\mathbf{U}}\, \underline{\mathbf{X}}^{b'} \right) \left(\underline{\mathbf{U}}\, \underline{\mathbf{X}}^{b'} \right)^{T} \circ \underline{\mathbf{L}} \right) \underline{\mathbf{U}}^{-T},$$

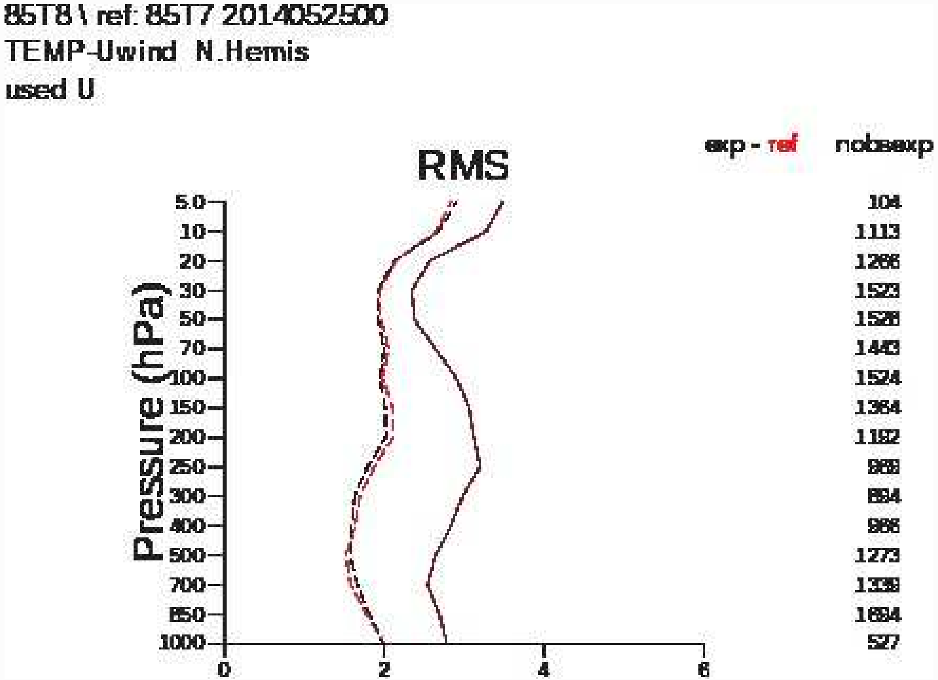
where $\underline{\mathbf{U}}$ is the change of variables.

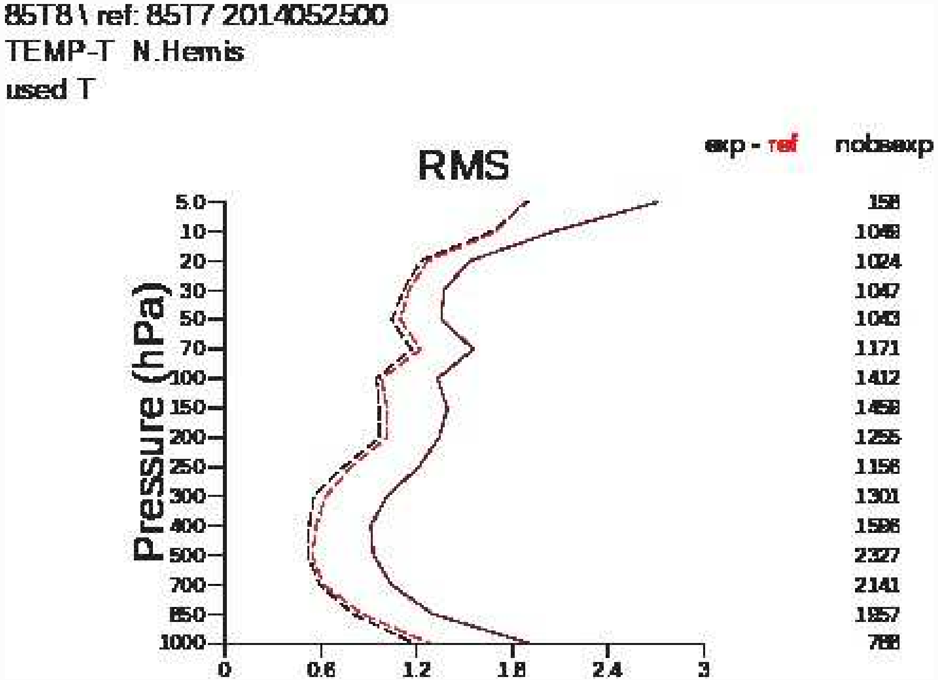
Optimization of horizontal localization length scales



$N^e = 200$

Results with conventional data
4D-Var / 4DEnVar ($N^e = 200$)





m/s °K

solid lines: RMS $y^o - H(x^b)$

dashed lines: RMS $y^o - H(x^a)$

Outline

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Advection of the localization

- Static $\underline{\mathbf{L}}$

$$\underline{\mathbf{L}} = \begin{pmatrix} \mathbf{L} & \mathbf{L} & & \mathbf{L} \\ \mathbf{L} & & & \\ & & & \\ \mathbf{L} & & & \mathbf{L} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{I} \\ \\ \mathbf{I} \end{pmatrix} \mathbf{L} \begin{pmatrix} \mathbf{I} & \mathbf{I} & & \mathbf{I} \end{pmatrix} = \underline{\mathbf{1}}\, \mathbf{L}\, \underline{\mathbf{1}}^{T} .$$

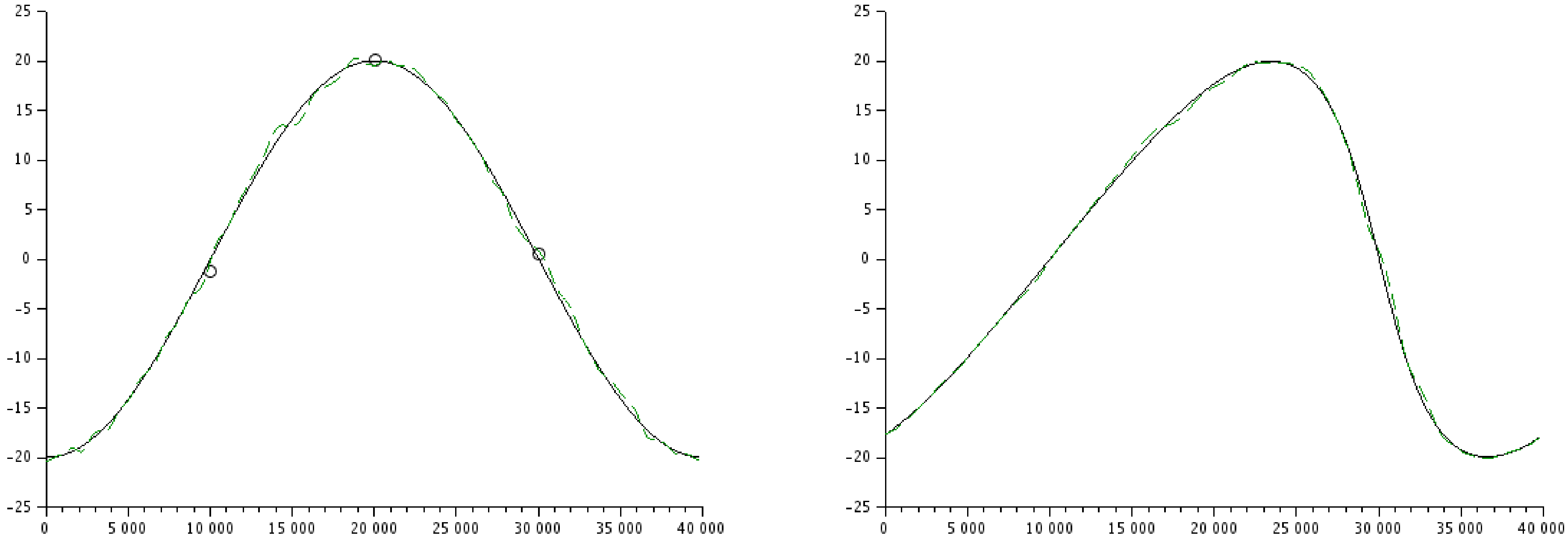
- Advected $\underline{\mathbf{L}}$

$$\underline{\mathbf{L}} = \begin{pmatrix} \mathbf{L} & \mathbf{L}\mathbf{A}_1^{T} & & \mathbf{L}\mathbf{A}_K^{T} \\ \mathbf{A}_1\mathbf{L} & & & \\ & & & \\ \mathbf{A}_K\mathbf{L} & & & \mathbf{A}_K\mathbf{L}\mathbf{A}_K^{T} \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{A}_1 \\ \\ \mathbf{A}_K \end{pmatrix} \mathbf{L} \begin{pmatrix} \mathbf{I} & \mathbf{A}_1^{T} & & \mathbf{A}_K^{T} \end{pmatrix} = \underline{\mathbf{A}}\, \mathbf{L}\, \underline{\mathbf{A}}^{T} .$$

Burgers'model

$$\partial u/\partial t + u\, \partial u/\partial x + \nu\, \partial^2 u/\partial x^2 = 0$$

m/s



t_0 km t_f=48h

____ u^t : true wind

----- u^b : simulated background

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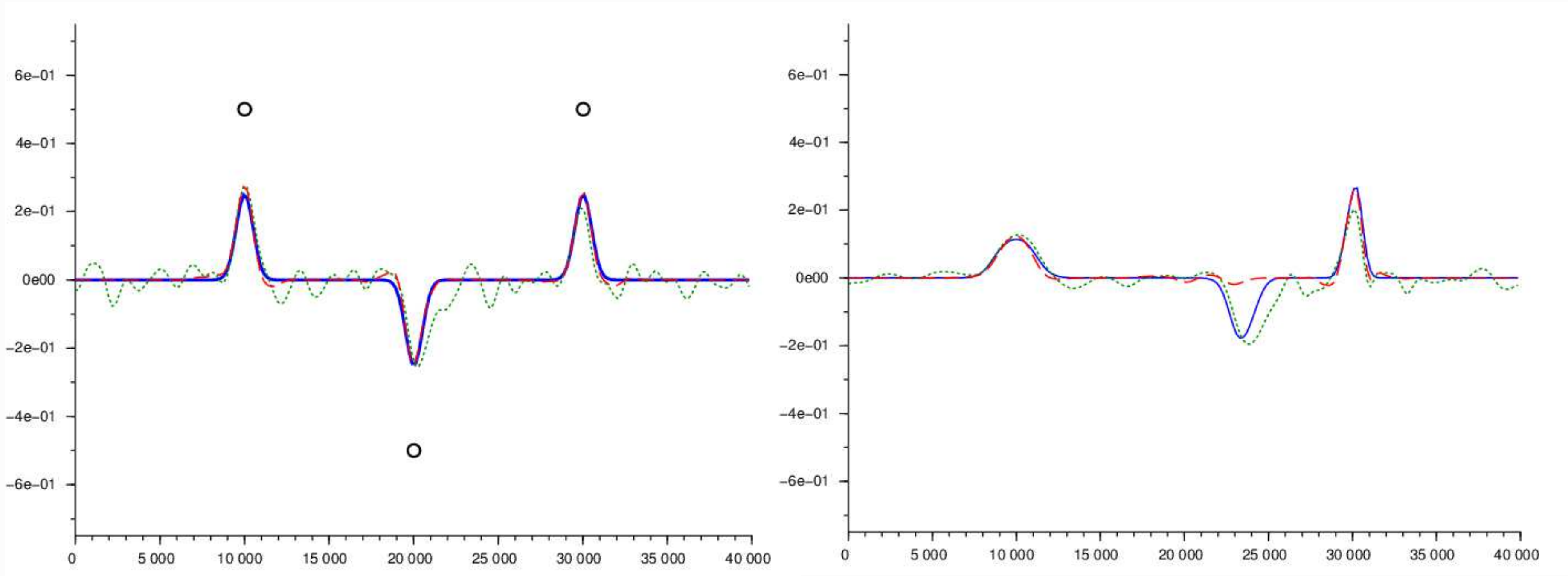
Static L

N^e = 100. Lc =1500 km

$$\underline{\mathbf{C}} = \underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T} \circ \underline{\mathbf{1}}\, \mathbf{L}\, \underline{\mathbf{1}}^T$$

m/s



t_0 km t_f=48h

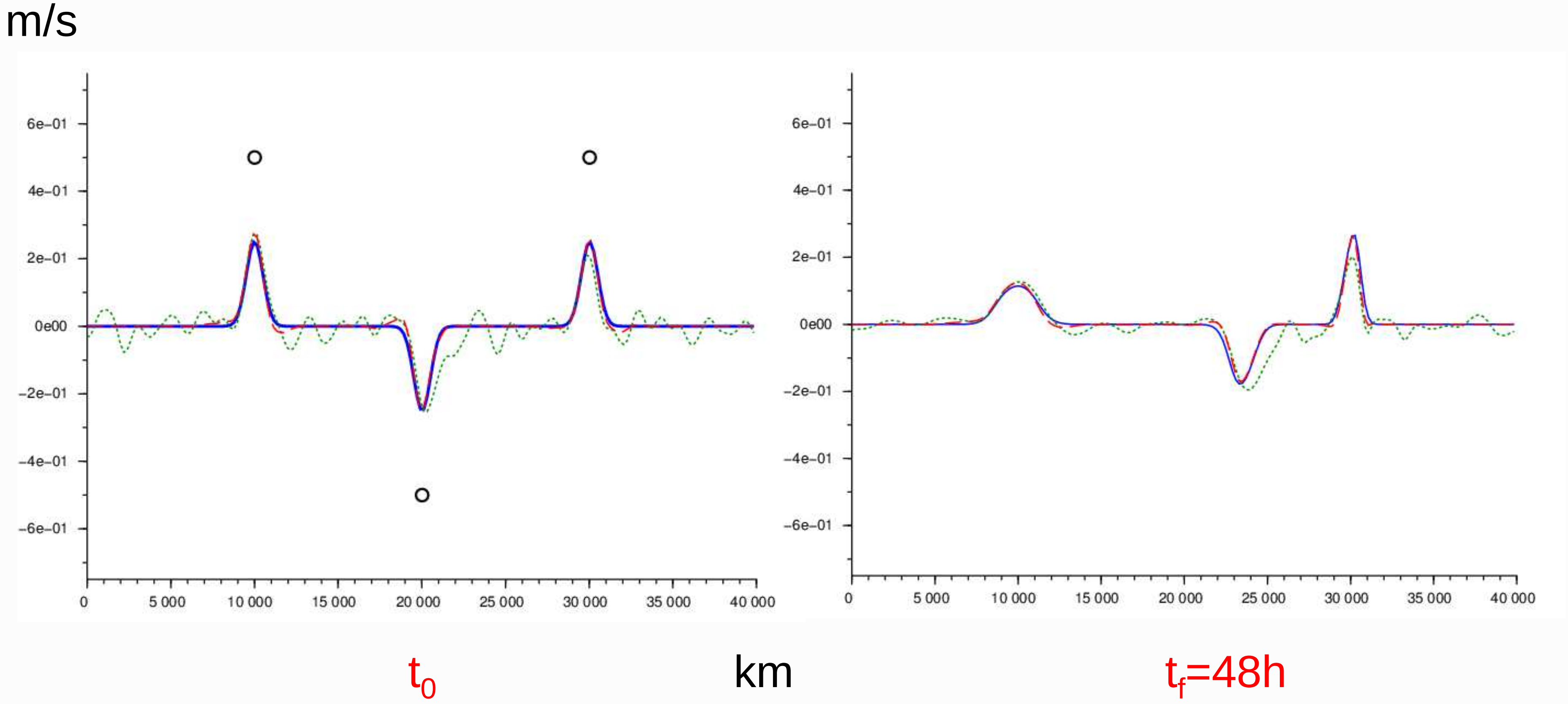
__ 4D-Var δu

…. 4DEnVar δu without localization

---- 4DEnVar δu with localization

Advected L
N^e = 100. Lc =1500 km

$$\underline{\mathbf{C}} = \underline{\mathbf{X}}^{b'} \underline{\mathbf{X}}^{b'T} \circ \underline{\mathbf{A}}\, \mathbf{L}\, \underline{\mathbf{A}}^T$$



__ 4D-Var δu

…. 4DEnVar δu without localization

---- 4DEnVar δu with adv localization

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Advection from t_0 to t_k
A Lagrangian point of view

- Lagrangian point of view

$$\alpha(s, t_k) = \alpha(s(t_0), t_0)$$

- Computation of deformed grid $s(t_0)$

$s(t_k\text{-}\delta t) = s$

for k' = k-1 : -1 : 0

$\quad s(t_{k'}\text{-}\delta t) = s(t_{k'}\text{-}\delta t) - u(s(t_{k'}\text{-}\delta t), t_{k'}\text{-}\delta t)) * \delta t$

end

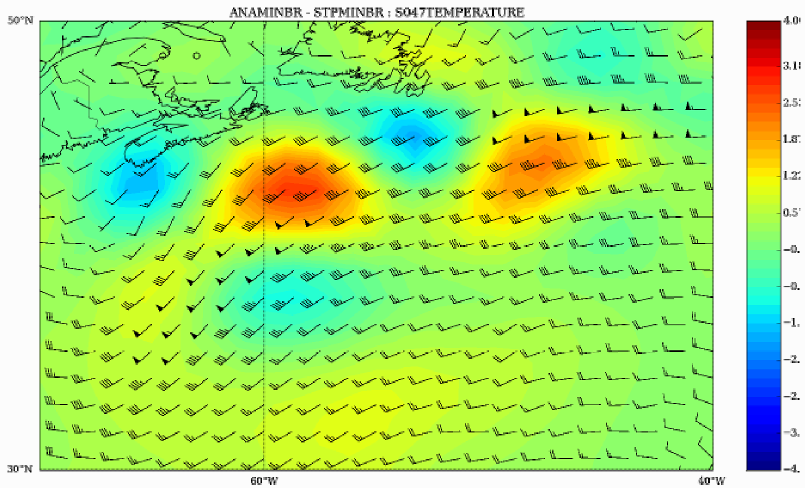
- Advection as a simple interpolation of $\alpha(t_0)$ at $s(t_0)$

$$\alpha(s, t_k) = \alpha(s(t_0), t_0) = I^{nt}\,(s(t_0))\ \alpha(s, t_0),$$

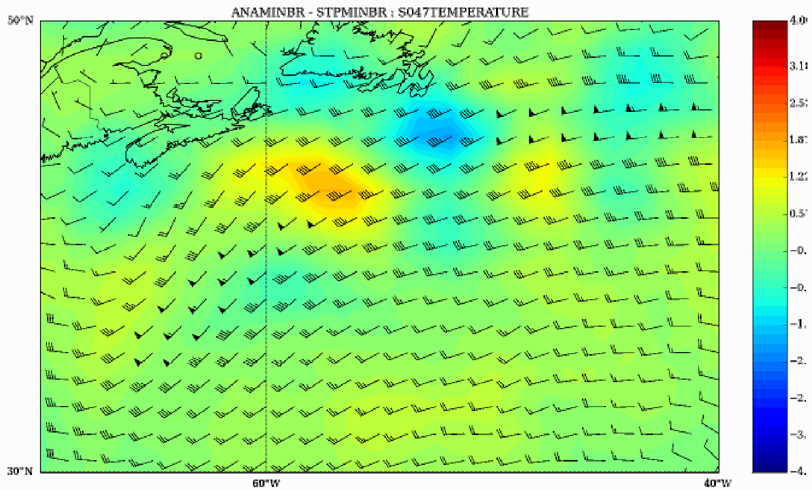
where $I^{nt}\,(s(t_0))$ are interpolation coefficients.

4D-Var / 4DEnVar

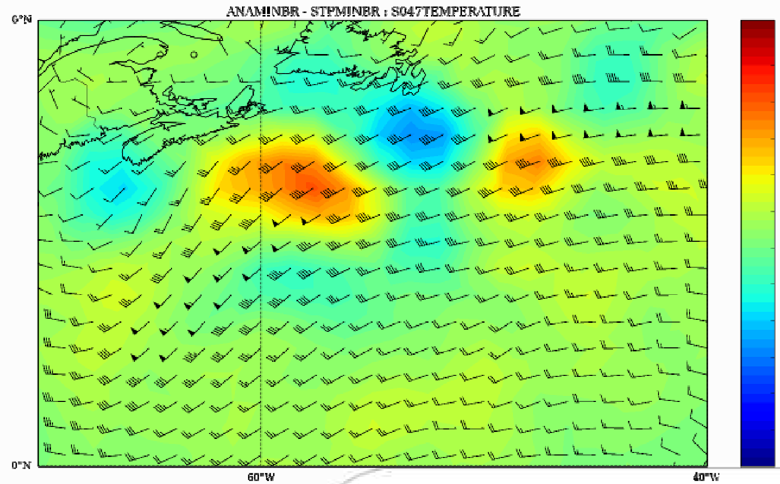
Temperature increment (°K) at t_0 with conv. obs. (@10 km)



4D-Var



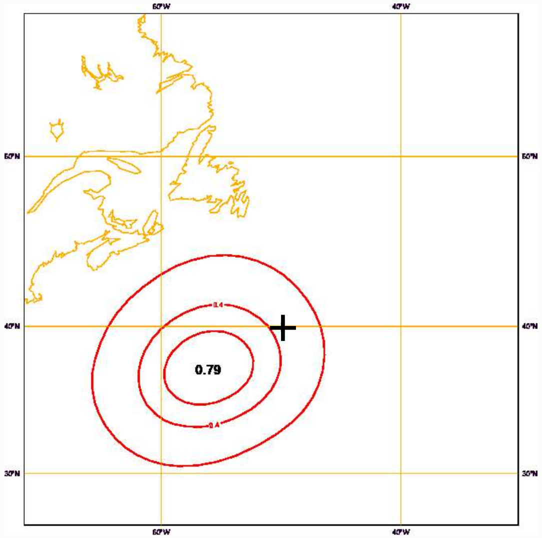
4DEnVar (N^e = 200)



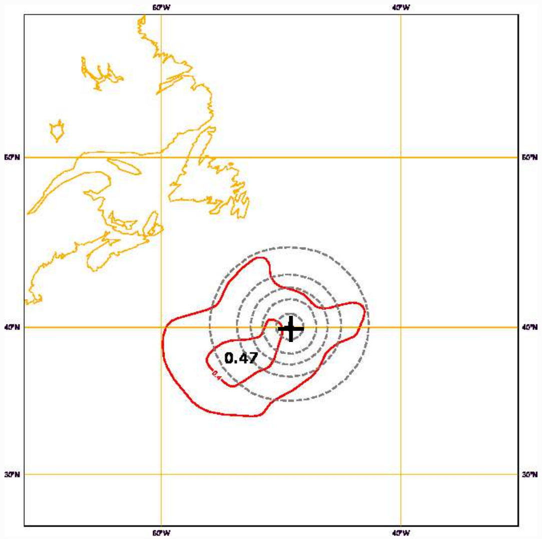
4DEnVar + localization advection

4D-Var / 4DEnVar

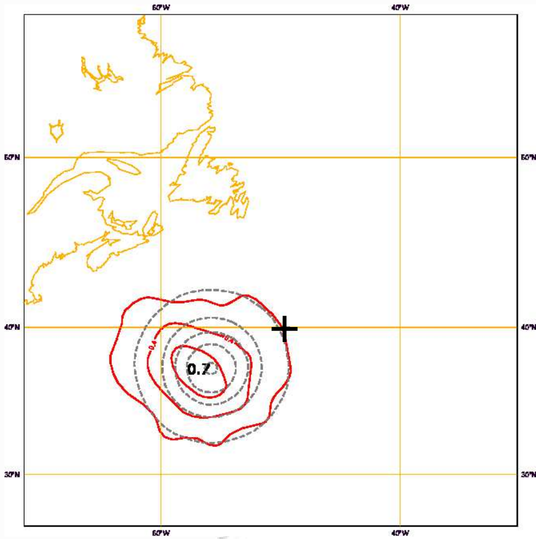
Temperature increment (°K) at t_0 with 1 obs at t_f (@10 km)



4D-Var



4DEnVar (N^e = 200)



4DEnVar + localization advection

Hybrid formulation

- Hybrid matrix: $\underline{\mathbf{B}}^h = \gamma^{c2}\, \underline{\mathbf{B}}^c + (1- \gamma^{c2})\, \underline{\mathbf{B}}^e.$

- Static $\underline{\mathbf{B}}^c$

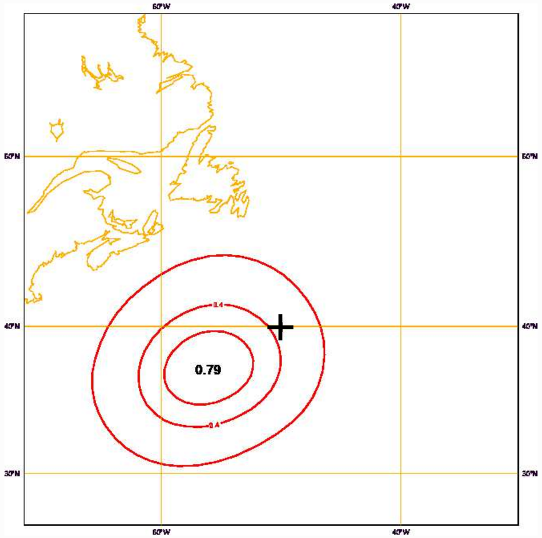
$$\underline{\mathbf{B}}^c = \begin{pmatrix} \mathbf{B}^c & \mathbf{B}^c & & \mathbf{B}^c \\ \mathbf{B}^c & & & \\ & & & \\ \mathbf{B}^c & & & \mathbf{B}^c \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{I} \\ \\ \mathbf{I} \end{pmatrix} \mathbf{B}^c \,(\,\mathbf{I} \;\; \mathbf{I} \qquad \mathbf{I}\,) = \underline{\mathbf{1}}\, \mathbf{B}^c\, \underline{\mathbf{1}}^T .$$

- Advected $\underline{\mathbf{B}}^c$

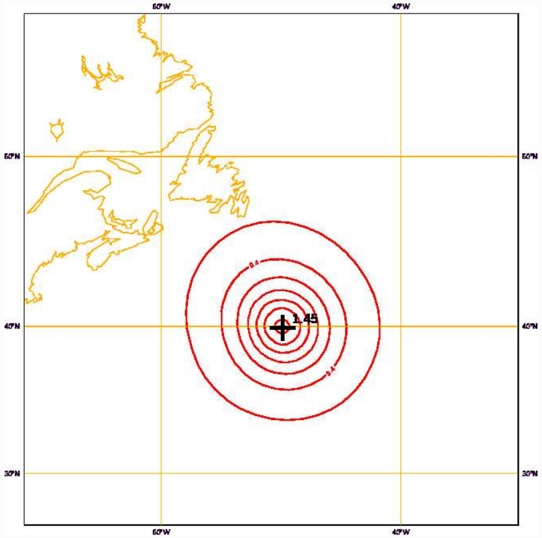
$$\underline{\mathbf{B}}^c = \begin{pmatrix} \mathbf{B}^c & \mathbf{B}^c\mathbf{A}_1^T & & \mathbf{B}^c\mathbf{A}_K^T \\ \mathbf{A}_1\mathbf{B}^c & & & \\ & & & \\ \mathbf{A}_K\mathbf{B}^c & & & \mathbf{A}_K\mathbf{B}^c\mathbf{A}_K^T \end{pmatrix} = \begin{pmatrix} \mathbf{I} \\ \mathbf{A}_1 \\ \\ \mathbf{A}_K \end{pmatrix} \mathbf{B}^c \,(\,\mathbf{I} \;\; \mathbf{A}_1^T \qquad \mathbf{A}_K^T\,) = \underline{\mathbf{A}}\, \mathbf{B}^c\, \underline{\mathbf{A}}^T .$$

4D-Var / 4DEnVar with B^c only (3D-Var FGAT)

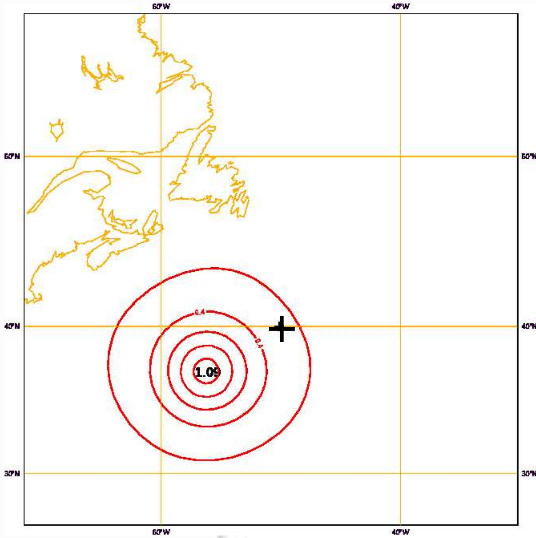
Temperature increment (°K) at t_0 with 1 obs at t_f (@10 km)



4D-Var



4DEnVar with static B^c



4DEnVar with advected B^c

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Conclusion and future work

- Conclusion

✓ Possible alternative to the use of α variables: 4D increment $\underline{\delta \mathbf{x}}$.

✓ Reading of perturbations is quick enough.

✓ First version of ARPEGE 4DEnVar at Météo-France.

✓ Localization is more difficult with the time dimension.

✓ The use of Lagrangian advection may help.

- Future work

✓ ARPEGE 4DEnVar: more observations, outer loops, Jc-DFI, …

✓ Improvement of spatial / spectral filtering, sensitivity to ensemble size.

✓ Ensemble of 4DEnVars.

✓ Development of 3D/4DEnVar for high resolution (1,3 km) LAM AROME.